Chapter 4

Nominal and Effective Interest Rates

Solutions to Problems

4.1 (a) monthly (b) quarterly (c) semiannually

4.2 (a) quarterly (b) monthly (c) weekly

4.3 (a) 12 (b) 4 (c) 2

4.4 (a) 1 (b) 4 (c) 12

4.5 (a) r/semi = 0.5*2 = 1% (b) 2% (c) 4%

4.6 (a) i = 0.12/6 = 2% per two months; r/4 months = 0.02*2 = 4%
(b) r/6 months = 0.02*3 = 6%
(c) r/2 yrs = 0.02*12 = 24%

4.7 (a) 5% (b) 20%

4.8 (a) effective (b) effective (c) nominal (d) effective (e) nominal

4.9 i/6months = 0.14/2 = 7%

4.10 $i = (1 + 0.04)^4 - 1$
$= 16.99\%$

4.11 $0.16 = (1 + r/2)^2 - 1$
$r = 15.41\%$

4.12 Interest rate is stated as effective. Therefore, i = 18%

4.13 $0.1881 = (1 + 0.18/m)^m - 1$
Solve for m by trial and gives m = 2

4.14 $i = (1 + 0.01)^2 - 1$
$i = 2.01\%$

4.15 i = 0.12/12 = 1% per month
Nominal per 6 months = 0.01(6) = 6%
Effective per 6 months = $(1 + 0.06/6)^6 - 1$
= 6.15%

4.16 (a) i/week = 0.068/26 = 0.262%
(b) effective

4.17 PP = weekly; CP = quarterly

4.18 PP = daily; CP = quarterly

4.19 From 2% table at n =12, F/P = 1.2682

4.20 Interest rate is effective
From 6% table at n = 5, P/G = 7.9345

4.21 P = 85(P/F,2%,12) = 85(0.7885)
= $67.02 million

4.22 F = 2.7(F/P,3%,60)
= 2.7(5.8916)
= $15.91 billion

4.23 P = 5000(P/F,4%,16)
= 5000(0.5339)
= $2669.50

4.24 P = 1.2(P/F,5%,1) (in $million)
= 1.2(0.9524)
= $1,142,880

4.25 P = 1.3(P/A,1%,28)(P/F,1%,2) (in $million)
= 1.3(24.3164)(0.9803)
= $30,988,577

4.26 F = 3.9(F/P,0.5%,120) (in $billion)
= 3.9(1.8194)
= $7,095,660,000

4.27 P = 3000(250 – 150)(P/A,4%,8) (in $million)
= 3000(100)(6.7327)
= $2,019,810

4.28 F = 50(20,000,000)(F/P,1.5%,9)
= 1,000,000,000(1.1434)
= $1.1434 billion

4.29 A = 3.5(A/P,5%,12) (in $million)
= 3.5(0.11283)
= $394,905

4.30 F = 10,000(F/P,4%,4) + 25,000(F/P,4%,2) + 30,000(F/P,4%,1)
= 10,000(1.1699) + 25,000(1.0816) + 30,000(1.04)
= $69,939

4.31 i/wk = 0.25%
P = 2.99(P/A,0.25%,40)
= 2.99(38.0199)
= $113.68

4.32 i/6 mths = $(1 + 0.03)^2 - 1$
A = 20,000(A/P,6.09%,4)
= 20,000 $\{[0.0609(1 + 0.0609)^4]/[(1 + 0.0609)^4-1]\}$
= 20,000(0.28919)
= $5784

4.33 F = 100,000(F/A,0.25%,8)(F/P,0.25%,3)
= 100,000(8.0704)(1.0075)
= $813,093

Subsidy = 813,093 – 800,000 = $13,093

4.34 P = (14.99 – 6.99)(P/A,1%,24)
= 8(21.2434)
= $169.95

4.35 First find P, then convert to A

P = 150,000$\{1 - [(1+0.20)^{10}/(1+0.07)^{10}\}]\}$/(0.07 – 0.20)
= 150,000(16.5197)
= $2,477,955

A = 2,477,955(A/P,7%,10)
= 2,477,955(0.14238)
= $352,811

4.36 P = 80(P/A,3%,12) + 2(P/G,3%,12)
P = 80(9.9540) + 2(51.2482)
= $898.82

4.37 2,000,000 = A(P/A,3%,8) + 50,000(P/G,3%,8)
2,000,000 = A(7.0197) + 50,000(23.4806)
A = $117,665

4.38 P = 1000 + 2000(P/A,1.5%,12) + 3000(P/A,1.5%,16)(P/F,1.5%,12)
= 1000 + 2000(10.9075) + 3000(14.1313)(0.8364)
= $58,273

4.39 First find P in quarter –1 and then use A/P to get A in quarters 0-8.
P_{-1} = 1000(P/F,4%,2) + 2000(P/A,4%,2)(P/F,4%,2) + 3000(P/A,4%,4)(P/F,4%,5)
= 1000(0.9246) + 2000(1.8861)(0.9246) + 3000(3.6299)(0.8219)
= $13,363

A = 13,363(A/P,4%,9)
= 13,363(0.13449)
= $1797.19

4.40 Move deposits to end of compounding periods and then find F.
F = 1800(F/A,3%,30)
= 1800(47.5754)
= $85,636

4.41 Move withdrawals to beginning of periods and then find F.
F = (10,000 – 1000)(F/P,4%,6) – 1000(F/P,4%,5) – 1000(F/P,4%,3)
= 9000(1.2653) – 1000(1.2167) – 1000(1.1249)
= $9046

4.42 Move withdrawals to beginning of periods and deposits to end; then find F.
F = 1600(F/P,4%,5) +1400(F/P,4%,4) – 2600(F/P,4%,3) + 1000(F/P,4%,2)
-1000(F/P,4%,1)
= 1600(1.2167) + 1400(1.1699) – 2600(1.1249) + 1000(1.0816) –1000(1.04)
= $701.44

4.43 Move monthly costs to end of quarter and then find F.
Monthly costs = 495(6)(2) = $5940
End of quarter costs = 5940(3) = $17,820
F = 17,820(F/A,1.5%,4)
= 17,820(4.0909)
= $72,900

4.44 $i = e^{0.13} - 1$
$= 13.88\%$

4.45 $i = e^{0.12} - 1$
$= 12.75\%$

4.46 $0.127 = e^{r} - 1$
r/yr = 11.96%
r /quarter = 2.99%

4.47 15% per year = 15/12 = 1.25% per month
$i = e^{0.0125} - 1 = 1.26\%$ per month

F = 100,000(F/A,1.26%,24)
$= 100{,}000\{[1 + 0.0126)^{24} - 1]/0.0126\}$
= 100,000(27.8213)
= $2,782,130

4.48 18% per year = 18/12 = 1.50% per month
$i = e^{0.015} - 1 = 1.51\%$ per month
P = 6000(P/A,1.51%,60)
$= 6000\{[(1 + 0.0151)^{60} - 1]/[0.0151(1 + 0.0151)^{60}]\}$
= 6000(39.2792)
= $235,675

4.49 $i = e^{0.02} - 1 = 2.02\%$ per month
A = 50(A/P,2.02%,36)
$= 50\{[0.0202(1 + 0.0202)^{36}]/[(1 + 0.0202)^{36} - 1]\}$
= 50(0.03936)
= $1,968,000

4.50 $i = e^{0.06} - 1 = 6.18\%$ per year
P = 85,000(P/F,6.18%,4)
$= 85{,}000[1/(1 + 0.0618)^{4}$
= 85,000(0.78674)
= $66,873

4.51 $i = e^{0.015} - 1 = 1.51\%$ per month
$2P = P(1 + 0.0151)^{n}$
$2.000 = (1.0151)^{n}$

Take log of both sides and solve for n
n = 46.2 months

4.52 Set up F/P equation in months.

$$3P = P(1 + i)^{60}$$
$$3.000 = (1 + i)^{60}$$
$$1.01848 = 1 + i$$
$$i = 1.85\% \text{ per month (effective)}$$

4.53 P = 150,000(P/F,12%,2)(P/F,10%,3)
= 150,000(0.7972)(0.7513)
= $89,840

4.54 F = 50,000(F/P,10%,4)(F/P,1%,48)
= 50,000(1.4641)(1.6122)
= $118,021

4.55 (a) First move cash flow in years 0-4 to year 4 at i = 12%.

F = 5000(F/P,12%,4) + 6000(F/A,12%,4)
= 5000(1.5735) + 6000(4.7793)
= $36,543

Now move the total to year 5 at i = 20%.

F = 36,543(F/P,20%,1) + 9000
= 36,543(1.20) + 9000
= $52,852

(b) Substitute A values for annual cash flows, including year 5 with the factor (F/P,20%,0) = 1.00

52,852 = A{[(F/P,12%,4) + (F/A,12%,4)](F/P,20%,1) + (F/P,20%,0)}
= A{[(1.5735) + (4.7793)](1.20) + 1.00}
= A(8.62336)

A = $6129 per year for years 0 through 5 (a total of 6 A values).

4.56 First find P.

P = 5000(P/A,10%,3) + 7000(P/A,12%,2)(P/F,10%3)
= 5000(2.4869) + 7000(1.6901)(0.7513)
= 12,434.50 + 8888.40
= $21,323

Now substitute A values for cash flows.

$$21{,}323 = A(P/A,10\%,3) + A(P/A,12\%,2)(P/F,10\%3)$$
$$= A(2.4869) + A(1.6901)(0.7513)$$
$$= A(3.7567)$$
$$A = \$5676$$

FE Review Solutions

4.57 Answer is (b)

4.58 Answer is (d)

4.59 $i/yr = (1 + 0.01)^{12} - 1 = 0.1268 = 12.68\%$
Answer is (d)

4.60 $i/quarter = e^{0.045} - 1 = 0.0460 = 4.60\%$
Answer is (c)

4.61 Answer is (d)

4.62 Answer is (a)

4.63 Find annual rate per year for each condition.
$i/yr = 22\%$ simple
$i/yr = (1 + 0.21/4)^4 - 1 = 0.2271 = 22.7\ \%$
$i/yr = (1 + 0.21/12)^{12} - 1 = 0.2314 = 23.14\ \%$
$i/yr = (1 + 0.22/2)^2 - 1 = 0.2321 = 23.21\ \%$

Answer is (a)

4.64 $i/semi\text{-}annual = e^{0.02} - 1 = 0.0202 = 2.02\%$
Answer is (b)

4.65 Answer is (c)

4.66 $P = 30(P/A,0.5\%,60)$
$= \$1552$
Answer is (b)

4.67 P = 7 + 7(P/A,4%,25)
= $116.3547 million
Answer is (c)

4.68 Answer is (a)

4.69 Answer is (d)

4.70 PP>CP; must use i over PP of 1 year. Therefore, n = 7
Answer is (a)

4.71 $P = 1{,}000{,}000 + 1{,}050{,}000\{[1- [(1 + 0.05)^{12}/(1 + 0.01)^{12}]\}/(0.01-0.05)$
= $16,585,447
Answer is (b)

4.72 Answer is (d)

4.73 Deposit in year 1 $= 1250/(1 + 0.05)^3$
= $1079.80
Answer is (d)

4.74 A = 40,000(A/F,5%,8)
= 40,000(0.10472)
= $4188.80
Answer is (c)

4.75 A = 800,000(A/P,3%,12)
= 800,000(0.10046)
= $80,368
Answer is (c)

Case Study Solution

1. Plan C:15-Year Rate - The calculations for this plan are the same as those for plan A, except that i = 9 ½% per year and n = 180 periods instead of 360. However, for a 5% down payment, the P&I is now $1488.04 which will yield a total payment of $1788.04. This is greater than the $1600 maximum payment available. Therefore, the down payment will have to be increased to $25,500, making the loan amount $124,500. This will make the P&I amount $1300.06 for a total monthly payment of $1600.06.

The amount of money required up front is now $28,245 (the origination fee has also changed). The plan C values for F_{1C}, F_{2C}, and F_{3C} are shown below.

$$F_{1C} = (40{,}000 - 28{,}245)(F/P,0.25\%,120)$$
$$= \$15{,}861.65$$

$$F_{2C} = 0$$

$$F_{3C} = 170{,}000 - [124{,}500(F/P,9.5\%/12,120) - 1300.06(F/A,9.5\%/12,120)]$$
$$= \$108{,}097.93$$

$$F_C = F_{1C} + F_{2C} + F_{3C}$$
$$= \$123{,}959.58$$

The future worth of Plan C is considerably higher than either Plan A ($87,233) or Plan B ($91,674). Therefore, Plan C with a 15-year fixed rate is the preferred financing method.

2. Plan A

Loan amount = $142,500
Balance after 10 years = $129,582.48
Equity = 142,500 – 129,582.48 = $12,917.52

Total payment made = 1250.56(120) = $150,067.20

Interest paid = 150,067.20 – 12,917.52 = $137,149.68

3. Amount paid through first 3 yrs = 1146.58 (36) = $41,276.88
Principal reduction through first 3 yrs = 142,500 – 139,297.08 = $3,202.92
Interest paid first 3 yrs = 41,276.88 – 3202.92 = $38,073.96

Amount paid year 4 = 1195.67(12) = 14,348.04
Principal reduction year 4 = 139,297.08 – 138,132.42 = 1164.66
Interest paid year 4 = 14,348.04 – 1164.66 = 13,183.38

Total interest paid in 4 years = 38,073.96 + 13,183.38 = $51,257.34

4. Let DP = down payment
Fixed fees = 300 + 200 + 200 + 350 + 150 + 300 = $1500
Available for DP = 40,000 – 1500 – (loan amount)(0.01)
where loan amount = 150,000 – DP

DP = 40,000 – 1500 - [(150,000 – DP)(0.01)]
= 40,000 – 1500 – 1500 + 0.01DP
0.99DP = 37,000
DP = $37,373.73

check: origination fee = (150,000 – 37,373.73)(0.01) = 1126.26
available DP = 40,000 – 1500 – 1126.26 = $37,373.73

5. Amount financed = $142,500

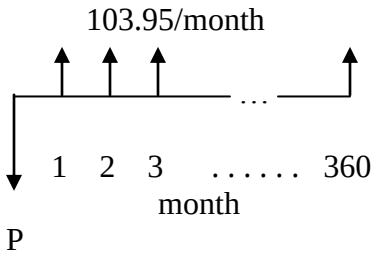
	Increase from one interest rate to the other
Monthly P&I @ 10% = $1,250.56	
Monthly P&I @ 11% = 142,500(A/P,11%/12, 60)	-----
$A = (142,500)\left[\frac{(0.009167)(1 + 0.009167)^{360}}{(1 + 0.009167)^{360} - 1}\right] = \$1357.06$	106.50
Monthly P&I @ 12% = $1465.77	108.71
Monthly P&I @ 13% = $1576.33	110.56
Monthly P&I @ 14% = $1688.44	112.11

Increase varies: 10% to 11% = $106.50
11% to 12% = 108.71
12% to 13% = 110.56
13% to 14% = 112.11

6. In buying down interest, you must give lender money now instead of money later. Therefore, to go from 10% to 9%, lender must recover the additional 1% now.

P&I @ 10% = 1250.54
P&I @ 9% = 1146.59

Difference = $103.95/month



P = 103.95(P/A,10%/12,360)
= 103.95(113.9508)
= $11,845.19