Chapter 18

Formalized Sensitivity Analysis and Expected Value Decisions

Solutions to Problems

18.1 <u>10 tons/day</u>

PW = –62,000 + 1500P/F,10%,8) – 0.50(10)(200)(P/A,10%,8) – 4(8)(200)(P/A,10%,8)
= –62,000 + 1500(0.4665) – 7400(5.3349)
= $–100,779

<u>20 tons/day</u>

PW = –62,000 + 1500(P/F,10%,8) – 0.50(20)(200)(P/A,10%,8)
– 8(8)(200)(P/A,10%,8)
= –62,000 + 1500(0.4665) – 14,800(5.3349)
= $–140,257

<u>30 tons/day</u>

Overtime hours required = (30/20)8 – 8 = 4.0 hours

PW = –62,000 + 1500(P/F,10%,8) – 0.50(30)(200)(P/A,10%,8)
– [8(8)(200) + 4.0(16)(200)](P/A,10%,8)
= –62,000 + 1500(0.4665) – 28,600(5.3349)
= $–213,878

18.2 Joe: PW = –77,000 + 10,000(P/F,8%,6) + 10,000(P/A,8%,6)
= –77,000 + 10,000(0.6302) + 10,000(4.6229)
= $–24,469

Jane: PW = –77,000 + 10,000(P/F,8%,6) + 14,000(P/A,8%,6)
= –77,000 + 10,000(0.6302) + 14,000(4.6229)
= $–5977

Carlos: PW = –77,000 + 10,000(P/F,8%,6) + 18,000(P/A,8%,6)
= –77,000 + 10,000(0.6302) + 18,000(4.6229)
= $12,514

Only the $18,000 revenue estimate of Carlos favors the investment.

18.3 Set up the spreadsheets for income estimates of \$10,000, 14,000 and 18,000 and calculate the PW at 8(1-0.35) = 5.2%. The \$18,000 revenue estimate is the only one with PW > 0.

	A	B	C	D	E	F	G	H
1	**Joe: \$10,000 = Revenue estimate**				MACRS	Taxable		
2	Year	Revenue	Expenses	P and S	Depreciation	income	Taxes	CFAT
3	0			-77000				\$ (77,000)
4	1	10000	2000		\$ 15,400	\$ (7,400)	\$ (2,590)	\$ 10,590
5	2	10000	2000		\$ 24,640	\$ (16,640)	\$ (5,824)	\$ 13,824
6	3	10000	2000		\$ 14,784	\$ (6,784)	\$ (2,374)	\$ 10,374
7	4	10000	2000		\$ 8,870	\$ (870)	\$ (305)	\$ 8,305
8	5	10000	2000		\$ 8,870	\$ (870)	\$ (305)	\$ 8,305
9	6	10000	2000	10000	\$ 4,435	\$ (6,435)	\$ (2,252)	\$ 20,252
10								
11	TI = B - C - D							
12	CFAT = B - C + D - G				PW of CFAT =	\$ (17,365)		
13	Depr. Recapture in year 6 is \$10,000							
14								
15	**Jane: \$14,000 = Revenue estimate**				MACRS	Taxable		
16	Year	Revenue	Expenses	P and S	Depreciation	income	Taxes	CFAT
17	0			-77000				\$ (77,000)
18	1	14000	2000		\$ 15,400	\$ (3,400)	\$ (1,190)	\$ 13,190
19	2	14000	2000		\$ 24,640	\$ (12,640)	\$ (4,424)	\$ 16,424
20	3	14000	2000		\$ 14,784	\$ (2,784)	\$ (974)	\$ 12,974
21	4	14000	2000		\$ 8,870	\$ 3,130	\$ 1,095	\$ 10,905
22	5	14000	2000		\$ 8,870	\$ 3,130	\$ 1,095	\$ 10,905
23	6	14000	2000	10000	\$ 4,435	\$ (2,435)	\$ (852)	\$ 22,852
24								
25					PW of CFAT =	\$ (4,252)		
26								
27	**Carlos: \$18,000 = Revenue estimate**				MACRS	Taxable		
28	Year	Revenue	Expenses	P and S	Depreciation	income	Taxes	CFAT
29	0			-77000				\$ (77,000)
30	1	18000	2000		\$ 15,400	\$ 600	\$ 210	\$ 15,790
31	2	18000	2000		\$ 24,640	\$ (8,640)	\$ (3,024)	\$ 19,024
32	3	18000	2000		\$ 14,784	\$ 1,216	\$ 426	\$ 15,574
33	4	18000	2000		\$ 8,870	\$ 7,130	\$ 2,495	\$ 13,505
34	5	18000	2000		\$ 8,870	\$ 7,130	\$ 2,495	\$ 13,505
35	6	18000	2000	10000	\$ 4,435	\$ 1,565	\$ 548	\$ 25,452
36								
37					PW of CFAT =	\$ 8,861		

18.4 $PW_{Build} = –80{,}000 – 70(1000) + 120{,}000(P/F,20\%,3)$
$= –150{,}000 + 120{,}000(0.5787)$
$= \$-80{,}556$

$PW_{Lease} = –(2.5)(12)(1000) – (2.50)(12)(1000)(P/A,20\%,2)$
$= –18{,}000 – 18{,}000(1.5278)$
$= \$–75{,}834$

The company should lease the space.

New construction cost = 70(0.90) = \$63 and lease at \$2.75

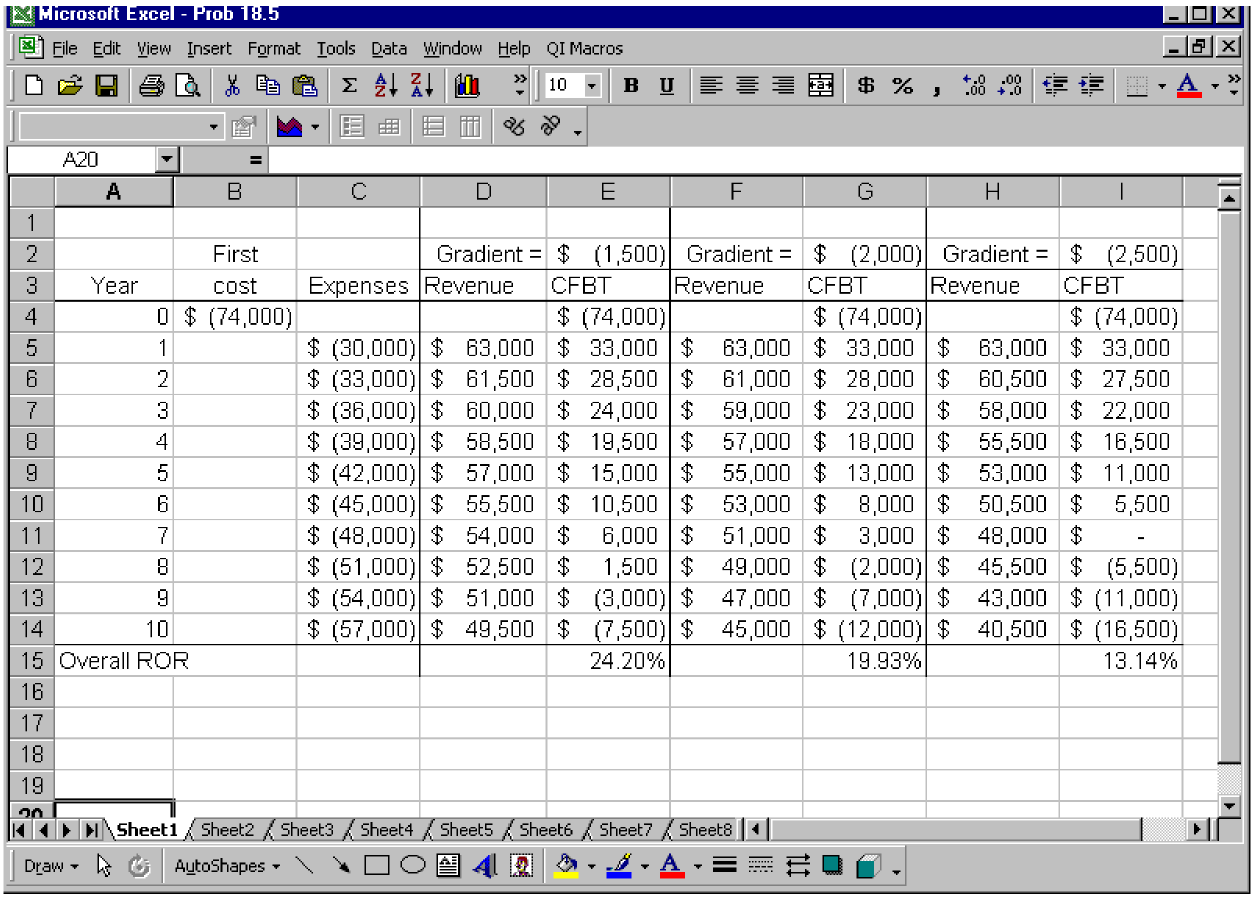
$PW_{Build} = –80{,}000 – 63(1000) + 120{,}000(P/F,20\%,3)$
$= –143{,}000 + 120{,}000(0.5787)$
$= \$–73{,}556$

$PW_{Lease} = –2.75(12)(1000)[1 + (P/A,20\%,2)]$
$= –15{,}000(2.5278)$
$= \$–83{,}417$

Select build, the decision is sensitive.

18.5 Calculate i* for G = \$1500, 2000 and 2500. Other gradient values can be used. All \$ values are in \$1000.

(a and b) Solution by Hand and Computer provide the same answers.



	A	B	C	D	E	F	G	H	I
1									
2		First		Gradient =	\$ (1,500)	Gradient =	\$ (2,000)	Gradient =	\$ (2,500)
3	Year	cost	Expenses	Revenue	CFBT	Revenue	CFBT	Revenue	CFBT
4	0	\$ (74,000)			\$ (74,000)		\$ (74,000)		\$ (74,000)
5	1		\$ (30,000)	\$ 63,000	\$ 33,000	\$ 63,000	\$ 33,000	\$ 63,000	\$ 33,000
6	2		\$ (33,000)	\$ 61,500	\$ 28,500	\$ 61,000	\$ 28,000	\$ 60,500	\$ 27,500
7	3		\$ (36,000)	\$ 60,000	\$ 24,000	\$ 59,000	\$ 23,000	\$ 58,000	\$ 22,000
8	4		\$ (39,000)	\$ 58,500	\$ 19,500	\$ 57,000	\$ 18,000	\$ 55,500	\$ 16,500
9	5		\$ (42,000)	\$ 57,000	\$ 15,000	\$ 55,000	\$ 13,000	\$ 53,000	\$ 11,000
10	6		\$ (45,000)	\$ 55,500	\$ 10,500	\$ 53,000	\$ 8,000	\$ 50,500	\$ 5,500
11	7		\$ (48,000)	\$ 54,000	\$ 6,000	\$ 51,000	\$ 3,000	\$ 48,000	\$ -
12	8		\$ (51,000)	\$ 52,500	\$ 1,500	\$ 49,000	\$ (2,000)	\$ 45,500	\$ (5,500)
13	9		\$ (54,000)	\$ 51,000	\$ (3,000)	\$ 47,000	\$ (7,000)	\$ 43,000	\$ (11,000)
14	10		\$ (57,000)	\$ 49,500	\$ (7,500)	\$ 45,000	\$ (12,000)	\$ 40,500	\$ (16,500)
15	Overall ROR				24.20%		19.93%		13.14%

For MARR = 18%, the decision does change from YES for G = \$1500 and \$2000, to NO for G = \$2500.

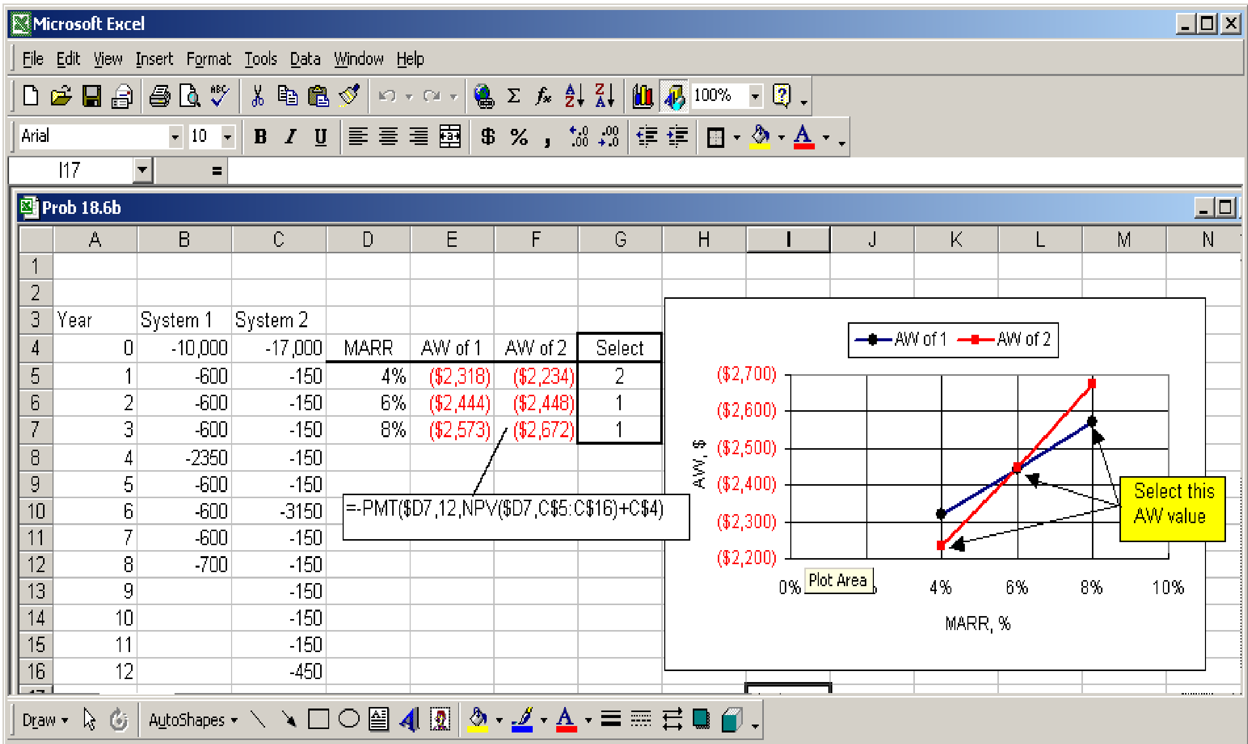
18.6 (a) The AW relations are:

$$AW_1 = -10{,}000(A/P,i,8) - 600 - 100(A/F,i,8) - 1750(P/F,i,4)(A/P,i,8)$$
$$AW_2 = -17{,}000(A/P,i,12) - 150 - 300(A/F,i,12) - 3000(P/F,i,6)(A/P,i,12)$$

Calculate AW for each MARR value. The decision is sensitive; it changes at 6%.

MARR	AW_1	AW_2	Select
4%	$–2318	$–2234	2
6	–2444	–2448	1
8	–2573	–2673	1

(b) Spreadsheet analysis: Use the PMT function to find AW over the life of each system.



18.7 (a) Breakeven number of vacation days per year is x.

$$AW_{cabin} = –130{,}000(A/P,10\%,10) + 145{,}000(A/F,10\%,10) – 1500 + 150x – (50/30)(1.20)x$$

$$AW_{trailer} = –75{,}000(A/P,10\%,10) + 20{,}000(A/F,10\%,10) – 1{,}750 + 125x – [300/30(0.6)](1.20)x$$

$$AW_{cabin} = AW_{trailer}$$

$$–130{,}000(0.16275) + 145{,}000(0.06275) – 1500 + 148x = –75{,}000(0.16275) + 20{,}000(0.06275) – 1750 + 105x$$

$-13{,}558.75 + 148x = -12{,}701.25 + 105x$

$43x = 857.5$

$x = 19.94$ days (Use $x = 20$ days per year)

(b) AW sensitivity analysis is performed for 12, 16, 20, 24, and 28 days.

$AW_{cabin} = -13{,}558.75 + 148x$
$AW_{trailer} = -12{,}701.25 + 105x$

Days, x	AW_{cabin}	$AW_{trailer}$	Selected
12	$-11,783	$-11,441	Trailer
16	-11,191	-11,021	Trailer
20	-10,599	-10,601	Cabin
24	-10,007	-10,181	Cabin
28	- 9415	- 9761	Cabin

Each pair of AW values are close to each other, especially for x = 20, which is the breakeven point.

(c) The trailer alternative. Select the alternative with the lower variable cost, since the variable term is positive, not a cost.

18.8 (a and b) Bond interest = b(50,000)/4 = $12,500(b), where b = 5%, 7%, and 9%. Use trial and error (a) or the IRR function (b) to find i* in the PW relation:

$$0 = -42{,}000 + (12{,}500b)(P/A,i^*,60) - 50{,}000(P/F,i^*,60)$$

Rate, b	Interest per quarter	i*/quarter	Nominal i* per year
5%	$625	1.67%	6.68%
7%	875	2.24	8.96
9%	1125	2.80	11.20

18.9 <u>6 years</u>

PW = –30,000 + 3500(P/A,8%,6) + 25,000(P/F,8%,6)
= –30,000 + 3500(4.6229) + 25,000(0.6302)
= $1935

<u>10 years</u>

PW = –30,000 + 3500(P/A,8%,10) + 15,000(P/F,8%,10)
= –30,000 + 3500(6.7101) + 15,000)0.4632)
= $433

<u>12 years</u>

PW = –30,000 + 3500(P/A,8%,12) + 8000(P/F,8%,12)
= $–447

The decision is sensitive to the life of the investment.

18.10 At i = 5%, find the AW value for n from 1 to 15.

AW = –8000(A/P,5%,n) – 500 – G(G/A,5%,n)

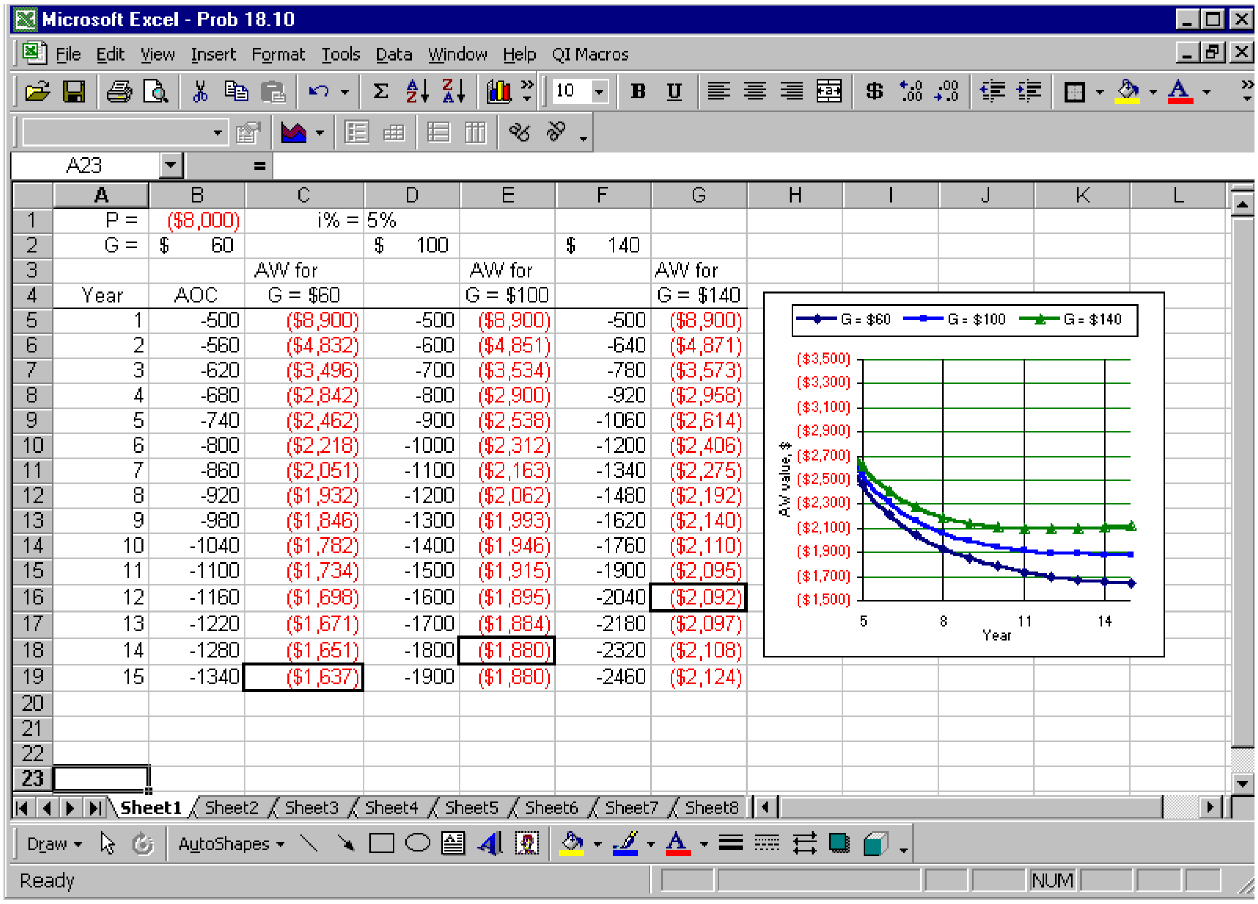
For spreadsheet analysis, use the PMT functions to obtain the AW for each n value for each G amount. The table below includes the analysis for G = $60, $100 and $140. As an example, the cell entries for G = $-60 are:

<u>For n = 1 year</u>

A5: 1
B5: $–500
C5: = -PMT(5%,1,NPV(5%,B5:B5)+B1)

<u>For n = 12 years</u>

A16: 12
B16: B15 – 60
C16: = -PMT(5%,12,NPV(5%,B5:B16)+B1)



	A	B	C	D	E	F	G
1	P =	($8,000)	i% =	5%			
2	G =	$ 60		$ 100		$ 140	
3			AW for		AW for		AW for
4	Year	AOC	G = $60		G = $100		G = $140
5	1	-500	($8,900)	-500	($8,900)	-500	($8,900)
6	2	-560	($4,832)	-600	($4,851)	-640	($4,871)
7	3	-620	($3,496)	-700	($3,534)	-780	($3,573)
8	4	-680	($2,842)	-800	($2,900)	-920	($2,958)
9	5	-740	($2,462)	-900	($2,538)	-1060	($2,614)
10	6	-800	($2,218)	-1000	($2,312)	-1200	($2,406)
11	7	-860	($2,051)	-1100	($2,163)	-1340	($2,275)
12	8	-920	($1,932)	-1200	($2,062)	-1480	($2,192)
13	9	-980	($1,846)	-1300	($1,993)	-1620	($2,140)
14	10	-1040	($1,782)	-1400	($1,946)	-1760	($2,110)
15	11	-1100	($1,734)	-1500	($1,915)	-1900	($2,095)
16	12	-1160	($1,698)	-1600	($1,895)	-2040	($2,092)
17	13	-1220	($1,671)	-1700	($1,884)	-2180	($2,097)
18	14	-1280	($1,651)	-1800	($1,880)	-2320	($2,108)
19	15	-1340	($1,637)	-1900	($1,880)	-2460	($2,124)

Results from columns C, E, and G are:

G	n*	AW
$60	15	$1637
100	14	1880
140	12	2092

The AW curves are quite flat; there are only a few dollars difference for the various n values around the n* value for each gradient value. The plot clearly shows this.

18.11 The PW relations are

$$PW_A = –P_A + (R_A – AOC_A)(P/A,20\%,5) + 50{,}000(P/F,20\%,5)$$

$$PW_B = –P_B + (R_B – AOC_B)(P/A,20\%,5) + 37{,}000(P/F,20\%,5)$$

Tabular results are presented below.

(a) First cost

	A		B	
Variation	Value	PW_A	Value	PW_B
–50%	$250,000	$–5610	$187,500	$–23,100
0.00	500,000	–255,610	375,000	–210,600
100%	1,000,000	–755,610	750,000	–585,600

(b) AOC

	A		B	
Variation	Value	PW_A	Value	PW_B
–50%	$37,500	$–143,463	$40,000	$–90,976
0.00	75,000	–255,610	80,000	–210,600
100%	150,000	–479,905	160,000	–449,848

(c) Revenue

	A		B	
Variation	Value	PW_A	Value	PW_B
–50%	$75,000	$–479,905	$65,000	$–404,989
0.00	150,000	–255,610	130,000	–210,600
100%	300,000	+192,980	260,000	+178,178

18.12 (a) Purchase price

Variation	Value, P	ROR	
–25%	$18,750	10.53%	
0.00	25,000	1.91%	(IRR function)
+25%	31,250	–4.47%	

$$0 = P – 5500(P/F,i,1) – 1500(P/F,i,2) – 1300(P/F,i,3) + 35,000(P/F,i,3)$$

Year	0	1	2	3
Cash flow, $	–P	–5500	–1500	33,700

(b) Selling price

Variation	Salvage, S	ROR	
–25%	$26,250	–8.74%	
0.00	35,000	1.91%	(IRR function)
+25%	43,750	10.83%	

0 = –25,000 –5500(P/F,i,1) – 1500(P/F,i,2) – 1300(P/F,i,3) + S(P/F,i,3)

Year	0	1	2	3
Cash flow, $	–25,000	–5500	–1500	S–1300

18.13 (a) First cost

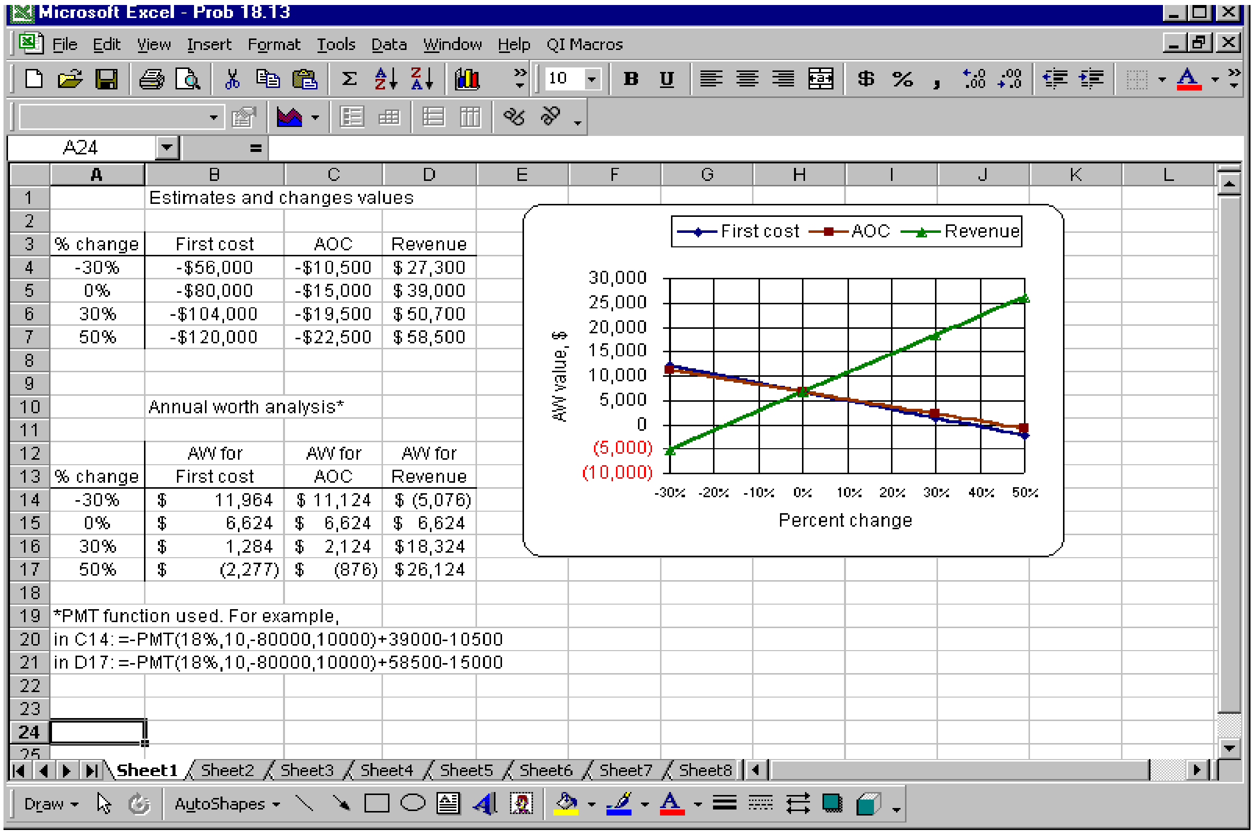
AW = –P(A/P,18%,10) + 10,000(A/F,18%,10) + 24,000
= –P(0.22251) + 24,425

(b) AOC

AW = –80,000(A/P,18%,10) + 10,000(A/F,18%,10) - AOC + 39,000
= –AOC + 21,624

(c) Revenue

AW = –80,000(A/P,18%,10) + 10,000(A/F,18%,10) – 15,000 + Revenue
= –32,376 + Revenue



Estimates and changes values

% change	First cost	AOC	Revenue
-30%	-$56,000	-$10,500	$ 27,300
0%	-$80,000	-$15,000	$ 39,000
30%	-$104,000	-$19,500	$ 50,700
50%	-$120,000	-$22,500	$ 58,500

Annual worth analysis*

% change	AW for First cost	AW for AOC	AW for Revenue
-30%	$ 11,964	$ 11,124	$ (5,076)
0%	$ 6,624	$ 6,624	$ 6,624
30%	$ 1,284	$ 2,124	$18,324
50%	$ (2,277)	$ (876)	$26,124

*PMT function used. For example,
in C14: =-PMT(18%,10,-80000,10000)+39000-10500
in D17: =-PMT(18%,10,-80000,10000)+58500-15000

18.14 PW calculates the amount you should be willing to pay now. Plot PW versus ± 30% changes in (a), (b) and (c) on one graph.

(a) Face value, P

$$\begin{aligned} PW &= P(P/F,4\%,20) + 450(P/A,4\%,20) \\ &= P(0.4564) + 6116 \end{aligned}$$

(b) Dividend rate, b

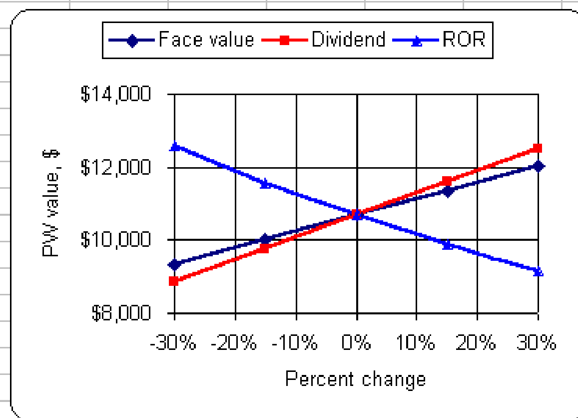
$$\begin{aligned} PW &= 10{,}000(P/F,4\%,20) + (10{,}000/2)(b)(P/A,4\%,20) \\ &= 10{,}000(0.4564) + b(5000)(13.5903) \\ &= 4564 + b(67{,}952) \end{aligned}$$

(c) Nominal rate, r

$$PW = 10{,}000(P/F,r,20) + 450(P/A,r,20)$$

Microsoft Excel - Prob 18.14

	A	B	C	D
6	% change	Face value	Dividend	ROR
7	-30%	$ 7,000	$ 315.00	2.8%
8	-15%	$ 8,500	$ 382.50	3.4%
9	0%	$ 10,000	$ 450.00	4.0%
10	15%	$ 11,500	$ 517.50	4.6%
11	30%	$ 13,000	$ 585.00	5.2%
12				
13		Present worth analysis*		
14				
15		PW for	PW for	PW for
16	% change	Face value	Dividend	ROR
17	-30%	$ 9,310	$ 8,845	$ 12,577
18	-15%	$ 9,995	$ 9,762	$ 11,578
19	0%	$ 10,680	$ 10,680	$ 10,680
20	15%	$ 11,364	$ 11,597	$ 9,871
21	30%	$ 12,049	$ 12,514	$ 9,142
22				
23	PV function used. For example:			
24	in B17: =-PV(4%,20,450,7000)			
25	in D21: =-PV(5.2%,20,450,10000)			



18.15 (a) 50 days Plan 1 - Purchase

Opt: $0.40 per ton (AOC = $2000)

$$AW = –6000(A/P,12\%,5) – 0.40(100)(50)$$
$$= –6000(0.27741) – 2000$$
$$= \$–3664$$

ML: $0.50 per ton (AOC = $2500)

$$AW = –6000(A/P,12\%,5) – 0.50(100)(50)$$
$$= –6000(0.27741) – 2,500$$
$$= \$–4164$$

18.15 (cont)

Pess: \$0.75 per ton (AOC = \$3750)

$$AW = –6000(A/P,12\%,5) – 0.75(100)(50)$$
$$= –6000(0.27741) – 3750$$
$$= \$–5414$$

Plan 2 - Lease

Opt: \$1800 lease

$$AW = –1800 – 50(8)(5.00) = \$–3800$$

ML: \$2500 lease

$$AW = –2500 – 50(8)(5.00) = \$–4500$$

Pess: \$3200 lease

$$AW = –3200 – 50(8)(5.00) = \$–5200$$

Plan 1 is better for the most likely estimates (\$0.50 and \$2500).

(b) 100 days

Plan 1 - Purchase

Opt: \$0.40 per ton (AOC = \$4000)

$$AW = –6000(A/P,12\%,5) – 0.40(100)(100)$$
$$= –6000(0.27741) – 4000$$
$$= \$–5664$$

ML: \$0.50 per ton (AOC = \$5000)

$$AW = –6000(A/P,12\%,5) – 0.50(100)(100)$$
$$= –6000(0.27741) – 5000$$
$$= \$–6664$$

Pess: 0.75 per ton (AOC = \$7500)

$$AW = –6000(A/P,12\%,5) – 0.75(100)(100)$$
$$= –6000(0.27741) – 7500$$
$$= \$–9164$$

Plan 2 - Lease

Opt: \$1800 lease

$$AW = –1800 – 100(8)(5.00) = \$–5800$$

ML: \$2,500 lease

$AW = –2500 – 100(8)(5.00) = \$–6500$

Pess: \$3,200 lease

$AW = –3200 – 100(8)(5.00) = \$–7200$

Plan 2 is better on the basis of the most likely estimates.

18.16 Water/wastewater cost = (0.12 + 0.04) per 1000 liters
= 0.16 per 1000 liters

Spray Method

Pessimistic - 100 liters

Water required = 10,000,000(100) = 1.0 billion

$AW = –(0.16/1000)(1.00 \times 10^9) = \$–160,000$

Most Likely - 80 liters

Water required = 10,000,000(80) = 800 million

$AW = –(0.16/1000)(800,000,000) = \$–128,000$

Optimistic - 40 liters

Water required = 10,000,000(40) = 400 million

$AW = –(0.16/1000)(400,000,000) = \$–64,000$

Immersion Method

$$AW = –10,000,000(40)(0.16/1000) – 2000(A/P,15\%,10) – 100$$
$$= –64,000 – 2000(0.19925) – 100$$
$$= \$–64,499$$

The immersion method is cheaper than the spray method, unless the optimistic estimate of 40 L is actually correct.

18.17 (a) MARR = 8% (Pessimistic)

PW_M = –100,000 + 15,000(P/A,8%,20)
= –100,000 + 15,000(9.8181)
= $47,272

PW_Q = –110,000 + 19,000(P/A,8%,20)
= –110,000 + 19,000(9.8181)
= $76,544

MARR = 10% (Most Likely)

PW_M = –100,000 + 15,000(P/A,10%,20)
= –100,000 + 15,000(8.5136)
= $27,704
PW_Q = –110,000 + 19,000(P/A,10%,20)
= –110,000 + 19,000(8.5136)
= $51,758

MARR = 15% (Optimistic)

PW_M = –100,000 + 15,000(P/A,15%,20)
= –100,000 + 15,000(6.2593)
= $–6111

PW_Q = –110,000 + 19,000(P/A,15%,20)
= –110,000 + 19,000(6.2593)
= $8927

(b) n = 16; Expanding economy (Optimistic)

n = 20(0.80) = 16 years

PW_M = –100,000 + 15,000(P/A,10%,16)
= –100,000 + 15,000(7.8237)
= $17,356

PW_Q = –110,000 + 19,000(P/A,10%,16)
= –110,000 + 19,000(7.8237)
= $38,650

n = 20; Expected economy (Most likely)

PW_M = \$27,704 (From part (a))

PW_Q = \$51,758 (From part (a))

n = 22; Receding economy (Pessimistic)

n = 20(1.10) = 22 years

PW_M = –100,000 + 15,000(P/A,10%,22)
= –100,000 + 15,000(8.7715)
= \$31,573

PW_Q = –110,000 + 19,000(P/A,10%,22)
= –110,000 + 19,000(8.7715)
= \$56,659

(c) Plot the PW values for each value of MARR and life. Plan M always has a lower PW value, so it is not accepted and plan Q is.

18.18 $E(flow_N)$ = 0.15(100) + 0.75(200) + 0.10(300)
= 195 barrels/day

$E(flow_E)$ = 0.35(100) + 0.15(200) + 0.45(300) + 0.05(400)
= 220 barrels/day

18.19 (a) E(time) = (1/4)(10 + 20 + 30 + 70) = 32.5 seconds
(b) E(time) = (1/3)(10 + 20 + 30) = 20 seconds

Yes, the 70 second estimate does increase the mean significantly.

18.20

n	1	2	3	4
Y	3	9	27	81

E(Y) = 3(0.4) + 9(0.3) + 27(0.233) + 81(0.067)
= 15.618

18.21 Solve for the low AOC from E(AOC)

$$E(AOC) = 4575 = 2800(0.25) + (\text{high AOC})(0.75)$$
$$\text{High AOC} = \$5167$$

18.22 $E(i) = 1/20[(-8)(1) + (-5)(1) + 0(5) + ... + 15(3)]$
$= 103/20 = 5.15\%$

18.23 $E(AW) = 0.15(300{,}000 – 25{,}000) + 0.7(50{,}000) = \$76{,}250$

18.24 (a) The subscripts identify the series by probability.

$$\begin{aligned} PW_{0.5} &= –5000 + 1000(P/A,20\%,3) \\ &= –5000 + 1000(2.1065) \\ &= \$–2894 \end{aligned}$$

$$\begin{aligned} PW_{0.2} &= –6000 + 500(P/F,20\%,1) + 1500(P/F,20\%,2) + 2000(P/F,20\%,3) \\ &= –6000 + 500(0.8333) + 1500(0.6944) + 2000(0.5787) \\ &= \$–3384 \end{aligned}$$

$$\begin{aligned} PW_{0.3} &= –4000 + 3000(P/F,20\%,1) + 1200(P/F,20\%,2) – 800(P/F,20\%,3) \\ &= –4000 + 3000(0.8333) + 1200(0.6944) – 800(0.5787) \\ &= \$–1130 \end{aligned}$$

$$\begin{aligned} E(PW) &= (PW_{0.5})(0.5) + (PW_{0.2})(0.2) + (PW_{0.3})(0.3) \\ &= –2894(0.5) – 3384(0.2) – 1130(0.3) \\ &= \$–2463 \end{aligned}$$

(b)
$$\begin{aligned} E(AW) &= E(PW)(A/P,20\%,3) \\ &= –2463(0.47473) \\ &= \$–1169 \end{aligned}$$

18.25 Determine E(AW) after calculating E(revenue).

$$\begin{aligned} E(\text{revenue}) &= 3[\text{no. days})(\text{no. climbers})(\text{income/climbers})](\text{probability}) \\ &= [(120)(350)(5)](0.3) + [(120)(350)(5) + 30(100)(5)](0.5) \\ &\quad + [(120\ )(350)(5) + (45)(100)(5)](0.2) \\ &= 63{,}000 + 112{,}500 + 46{,}500 \\ &= \$222{,}000 \end{aligned}$$

$$E(AW) = –375{,}000(A/P,12\%,10) – 25{,}000[(P/F,12\%,4) + (P/F,12\%,8)](A/P,12\%,10) – 56{,}000 + 222{,}000$$
$$= –375{,}000(0.17698) – 25{,}000[(0.6355) + (0.4039)](0.17698) + 166{,}000$$
$$= \$95{,}034$$

The mock mountain should be constructed.

18.26 Determine E(PW) after calculating the PW of E(revenue).

E(revenue) = P(slump)(revenue over 3-year periods)

PW(E(revenue)) = PW [P(slump)(revenue 1st 3 years)
+P(slump) (revenue 2nd 3 years)
+ P(expansion)(revenue 1st 3 years)
+ P(expansion)(revenue 2nd 3 years)]

$$= 0.5[20{,}000(P/A,8\%,3)] + 0.2[20{,}000(P/A,8\%,3)(P/F,8\%,3)] + 0.5[35{,}000(P/A,8\%,3)] + 0.8[35{,}000(P/A,8\%,3)(P/F,8\%,3)]$$

$$= 0.5[51{,}542] + 0.2\,[40{,}914] + 0.5\,[90{,}198] + 0.8\,[71{,}600]$$

$$= \$136{,}333$$

$$E(PW) = –200{,}000 + 200{,}000(0.12)\,(P/F,8\%,6) + PW(E(revenue))$$
$$= –200{,}000 + 15{,}125 + 136{,}333$$
$$= \$–48{,}542$$

No, less than an 8% return is expected.

18.27 Certificate of Deposit

Rate of return = 6.35% (from problem statement)

Stocks

Stock 1: – 5000 + 250(P/A,i%,4) + 6800(P/F,i%,5) = 0
is the i* relation.
i* = 10.07% (RATE function)

Stock 2: –5000 + 600(P/A,i%,4) + 4000(P/F,i%,5) = 0
i* = 6.36% (RATE function)

E(i) = 10.07(0.5) + 6.36(0.5) = 8.22%

Real Estate

Rate of return with Prob = 0.3
–5,000 – 425(P/A,i%,4) + 9500(P/F,i%,5) = 0
i* = 8.22%

Rate of return with Prob. 0.5
–5000 + 7200(P/F,i%,5) = 0
(P/F,i%,5) = 0.6944
i* = 7.57%

Rate of return with Prob. 0.2
–5000 + 500(P/A,i%,4) + 100(P/G,i%,4) + 5200(P/F,i%,5) = 0
i* = 11.34%

E(i) = 8.22(0.3) + 7.57(0.5) + 11.34(0.2)
= 8.52%

Invest in real estate for the highest E(rate of return) of 8.52%.

18.28 (a) Calculate fraction in equity times i on equity from graph.

E(i) = 0.3(i on 20-80) + 0.5(i on 50-50) + 0.2(i on 80-20)
= 0.3(7%) + 0.5(9%) + 0.2(11.5)
= 8.9%

(b) (Fraction of pool)($1 million)(fraction of D-E in equity)

Amount = 0.3($1 mil)(0.8) + 0.5($1 mil)(0.5) + 0.2($1 mil)(0.2)
= 0.3 (800,000) + 0.5(500,000) + 0.2(200,000)
= 240,000 + 250,000 + 40,000
= $530,000

The FW is calculated using the correct i rate for each equity amount.

FW = 240,000(F/P,7%,10) + 250,000(F/P,9%,10) + 40,000(F/P,11.5%,10)

$= 240{,}000(1.9672) + 250{,}000(2.3674) + 40{,}000(2.9699)$
$= \$1{,}182{,}755$

(c) Use Eq. [14.9] to determine the real i. The graph rates are actually i_f values.

at $i_f = 7\%$: $i = (i_f - f)/(1 + f)$
$= (0.07 - 0.045)/(1 + 0.045)$
$= 0.0239$ (2.39%)
at $i_f = 9\%$: $i = (0.09 - 0.045)/1.045 = 0.043$ (4.3%)

at $i_f = 11.5\%$: $i = (0.115 - 0.045)/1.045 = 0.067$ (6.7%)

This is case 2 in Sec. 14.3. Use Eq. [14.8].

$FW = 1{,}182{,}755/(1.045)^{10}$
$= 1{,}182{,}755/1.55297$
$= \$761{,}608$

Alternatively, find FW at the real i for each equity amount.

FW $= 240{,}000(F/P,2.39\%,10) + 250{,}000(F/P,4.3\%,10)$
$+40{,}000(F/P,6.7\%,10)$
$= 240{,}000(1.26641) + 250{,}000(1.5238) + 40{,}000(1.9127)$
$= \$303{,}939 + 380{,}876 + 76{,}508$
$+ \$761{,}323$ (Rounding of i makes the difference)

18.29 AW = annual loan payment + (damage) x P(rainfall amount or greater)

The subscript on AW indicates the rainfall amount.

$AW_{2.0} = -200{,}000(A/P,6\%,10) + (-50{,}000)(0.3)$
$= -200{,}000(0.13587) -50{,}000(0.3)$
$= \$-42{,}174$

$AW_{2.25} = -225{,}000(A/P,6\%,10) + (-50{,}000)(0.1)$
$= -300{,}000(0.13587) -50{,}000(0.1)$
$= \$-35{,}571$

$AW_{2.5} = -300{,}000(A/P,6\%,10) + (-50{,}000)(0.05)$

$= –350,000(0.13587) –50,000(0.05)$
$= \$–43,261$

$AW_{3.0} = –400,000(A/P,6\%,10) + (–50,000)(0.01)$
$= –400,000(0.13587) –50,000(0.01)$
$= \$–54,848$

$AW_{3.25} = –450,000(A/P,6\%,10) + (–50,000)(0.005)$
$= –450,000(0.13587) –50,000(0.005)$
$= \$–61,392$

Build a wall to protect against a rainfall of 2.25 inches with an expected AW of \$–35,571.

18.30 Compute the expected value for each outcome and select the minimum for D3.

Top node: 0.2(55) + 0.35(–30) + 0.45(10) = 5.0

Bottom node: 0.4(–17) + 0.6(0) = –6.8

Indicate 5.0 and –6.8 in ovals and select the top branch with E(value) = 5.0.

18.31 Maximize the value at each decision node.

D3: Top: E(value) = \$30
Bottom: E(value) = 0.4(100) + 0.6(–50) = \$10

Select top at D3 for \$30

D1: Top: 0.9(D3 value) + 0.1(final value)
0.9(30) + 0.1(500) = \$77
Value at D1 = 77-50 = \$27
Bottom: 90 – 80 = \$10

Select top at D1 for \$27

D2: Top: E(value) = 0.3(150 – 30) + 0.4(75) = \$66
Middle: E(value) = 0.5(200 – 100) = \$50
Bottom: E(value) = \$50

At D2, value = E(value) – investment

Top: $66 - 25 = \$41$ (maximum)
Middle: $50 - 30 = \$20$
Bottom: $50 - 20 = \$30$

Select top at D2 for $41

Conclusion: Select D2 path and choose top branch ($25 investment)

18.32 Calculate the E(PW) in year 3 and select the largest expected value. In $1000 terms:

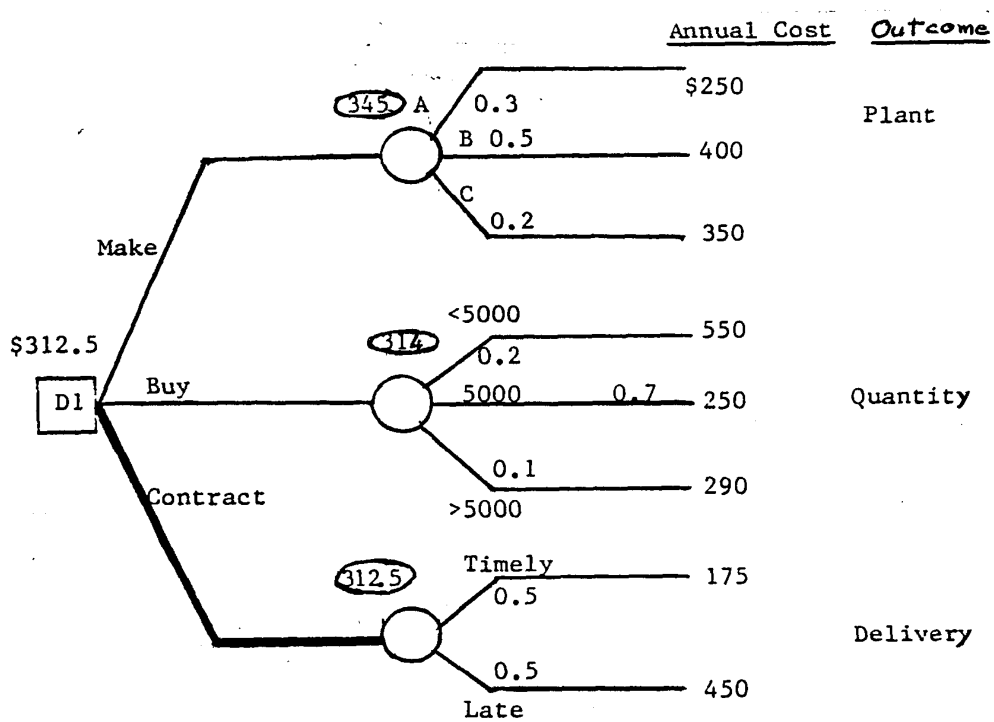
$$\begin{aligned} E(\text{PW of D4},x) &= -200 + 0.7[50(P/A,15\%,3)] + 0.3[40(P/F,15\%,1) \\ &\quad +30(P/F,15\%,2) + 20(P/F,15\%,3)] \\ &= -98.903 \qquad (\$-98{,}903) \end{aligned}$$

$$\begin{aligned} E(\text{PW of D4},y) &= -75 + 0.45[30(P/A,15\%,3) + 10(P/G,15\%,3)] \\ &\quad +0.55[30(P/A,15\%,3)] \\ &= 2.816 \qquad (\$2816) \end{aligned}$$

$$\begin{aligned} E(\text{PW of D4},z) &= -350 + 0.7[190(P/A,15\%,3) - 20(P/G,15\%,3)] \\ &\quad + 0.3[-30(P/A,15\%,3)] \\ &= -95.880 \qquad (\$-95{,}880) \end{aligned}$$

Select decision branch y; it has the largest E(PW).

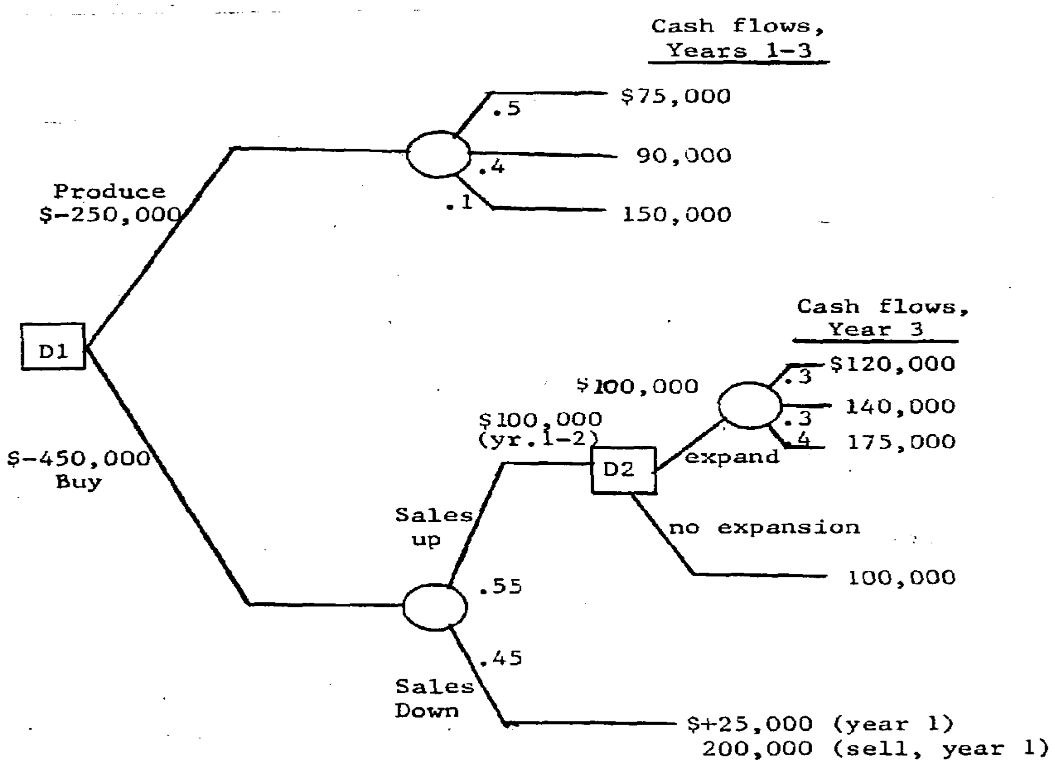
18.33 Select the minimum E(cost) alternative. (All dollar values are times $-1000).



Make:	E(cost of plant) = 0.3(250) + 0.5(400) + 0.2(350)
	= $345 ($345,000)
Buy:	E(cost of quantity) = 0.2(550) + 0.7(250) + 0.1(290)
	= $314 ($314,000)
Contract:	E(cost of delivery) = 0.5(175 + 450)
	= $312.5 ($312,500)

Select the contract alternative since the E(cost of delivery) is the lowest at $-312,500.

18.34 (a) Construct the decision tree.



(b) At D2 compute PW of cash flows and E(PW) using probability values.

<u>Expansion option</u>

(PW for D2, $120,000) = –100,000 + 120,000(P/F,15%,1)
= $4352

(PW for D2, $140,000) = –100,000 + 140,000(P/F,15%,1)
= $21,744

(PW for D2, $175,000) = $52,180

E(PW) = 0.3(4352 + 21,744) + 0.4(52,180) = $28,700

18.34 (cont) No expansion option

(PW for D2, \$100,000 = \$100,000(P/F,15%,1) = \$86,960

E(PW) = \$86,960

Conclusion at D2: Select no expansion option

(c) Complete foldback to D1 considering 3 year cash flow estimates.

Produce option, D1

E(PW of cash flows) = [0.5(75,000) + 0.4(90,000) + 0.1(150,000](P/A,15%,3)
= \$202,063

E(PW for produce) = cost + E(PW of cash flows)
= –250,000 + 202,063
= \$–47,937

Buy option, D1

At D2, E(PW) = \$86,960

E(PW for buy) = cost + E(PW of sales cash flows)
= –450,000 + 0.55(PW sales up) + 0.45(PW sales down)

PW Sales up = 100,000(P/A,15%,2) + 86,960(P/F,15%,2)
= \$228,320

PW sales down = (25,000 + 200,000)(P/F,15%,1)
= \$195,660

E(PW for buy) = –450,000 + 0.55 (228,320) + 0.45(195,660)
= \$–236,377

Conclusion: E(PW for produce) is larger than E(PW for buy); select produce option.

Note: The returns are both less than 15%, but the return is larger for produce option than buy.

(d) The return would increase on the initial investment, but would increase faster for the produce option.

Extended Exercise Solution

1. Relations are developed here for hand solution.

<u>MARR = 8%</u>

$$
\begin{aligned}
PW_A &= -10,000 + 1000(P/F,8\%,40) - 500(P/A,8\%,40) \\
&= -10,000 + 1000(0.0460) - 500(11.9246) \\
&= \$-15,916
\end{aligned}
$$

$$
\begin{aligned}
PW_B &= -30,000 + 5000(P/F,8\%,40) - 100(P/A,8\%,40) - 5000 \\
&\quad - 200(P/F,8\%,20) - 5000(P/F,8\%,20) - 200(P/F,8\%,40) - 200(P/A,8\%,40) \\
&= -35,000 + 4800(P/F,8\%,40) - 300(P/A,8\%,40) - 5200(P/F,8\%,20) \\
&= -35,000 + 4800(0.0460) - 300(11.9246) - 5200(0.2145) \\
&= \$-39,472
\end{aligned}
$$

<u>MARR = 10%</u>

$$
\begin{aligned}
PW_A &= -10,000 + 1000(P/F,10\%,40) - 500(P/A,10\%,40) \\
&= -10,000 + 1000(0.0221) - 500(9.7791) \\
&= \$-14,867
\end{aligned}
$$

$$
\begin{aligned}
PW_B &= -30,000 + 5000(P/F,10\%,40) - 100(P/A,10\%,40) - 5000 \\
&\quad - 200(P/F,10\%,20) - 5000(P/F,10\%,20) - 200(P/F,10\%,40) \\
&\quad - 200(P/A,10\%,40) \\
&= -35,000 + 4800(P/F,10\%,40) - 300(P/A,10\%,40) - 5200(P/F,10\%,20) \\
&= -35,000 + 4800(0.0221) - 300(9.7791) - 5200(0.1486) \\
&= \$-38,600
\end{aligned}
$$

<u>MARR = 15%</u>

$$
\begin{aligned}
PW_A &= -10,000 + 1000(P/F,15\%,40) - 500(P/A,15\%,40) \\
&= -10,000 + 1000(0.0037) - 500(6.6418) \\
&= \$-13,317
\end{aligned}
$$

$$
\begin{aligned}
PW_B &= -30,000 + 5000(P/F,15\%,40) - 100(P/A,15\%,40) - 5000 \\
&\quad -200(P/F,15\%,20) - 5000(P/F,15\%,20) - 200(P/F,15\%,40) \\
&\quad -200(P/A,15\%,40) \\
&= -35,000 + 4800(P/F,15\%,40) - 300(P/A,15\%,40) - 5200(P/F,15\%,20) \\
&= -35,000 + 4800(0.0037) - 300(6.6418) - 5200(0.0611) \\
&= \$-37,293
\end{aligned}
$$

Not very sensitive.

2. Expanding economy

n_A = 40(0.80) = 32 years
n_1 = 40(0.80) = 32 years
n_2 = 20(0.80) = 16 years

PW_A = –10,000 + 1000(P/F,10%,32) – 500(P/A,10%,32)
= –10,000 + 1,000(0.0474) – 500(9.5264)
= $–14,716

PW_B = –30,000 + 5000(P/F,10%,32) – 100(P/A,10%,32) – 5000
–200(P/F,10%,16) – 5000(P/F,10%,16) – 200(P/F,10%,32)
–200(P/A,10%,32)
= –35,000 + 4800(P/F,10%,32) – 300(P/A,10%,32) – 5200(P/F,10%,16)
= –35,000 + 4800(0.0474) – 300(9.5264) – 5200(0.2176)
= $–38,762

Expected economy

PW_A = $–14,876 (from #1)

PW_B = $–38,600 (from #1)

Receding economy

n_A = 40(1.10) = 44 years
n_1 = 40(1.10) = 44 years
n_2 = 20(1.10) = 22 years

PW_A = –10,000 + 1000(P/F,10%,44) – 500(P/A,10%,44)
= –10,000 + 1000(0.0154) – 500(9.8461)
= $–14,908

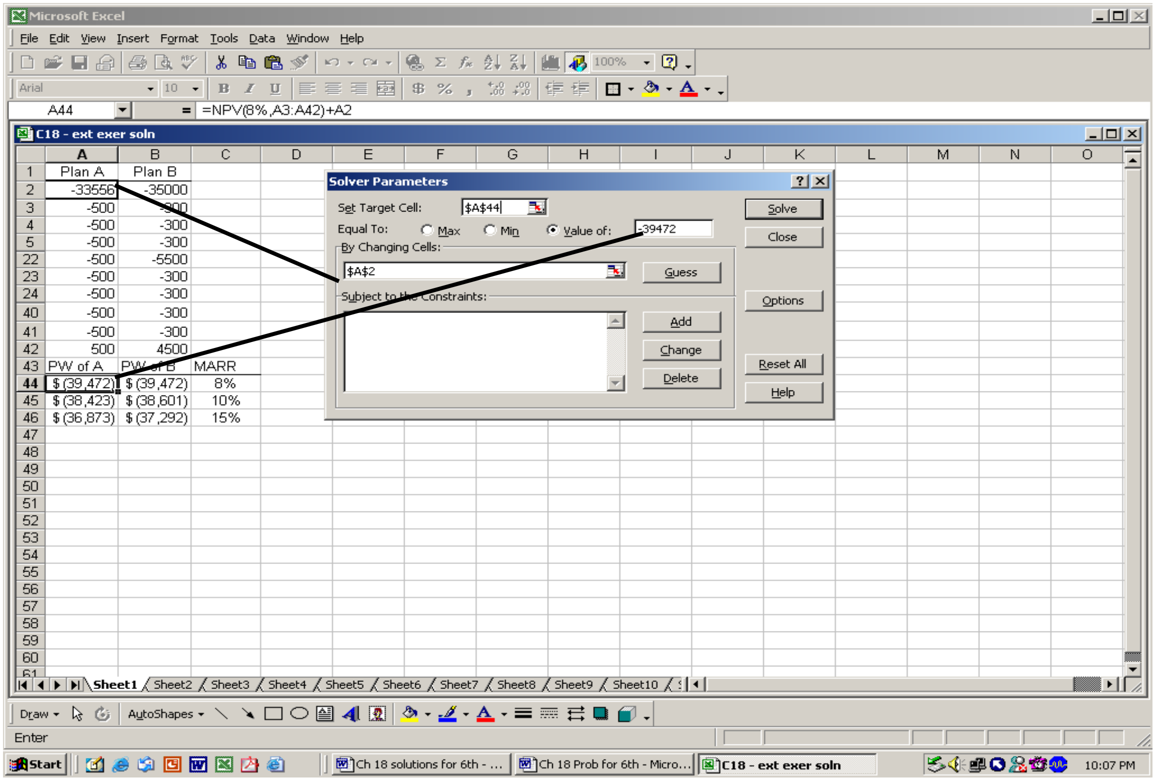
PW_B = –30,000 + 5000(P/F,10%,44) – 100(P/A,10%,44) – 5000
– 200(P/F,10%,22) – 5000(P/F,10%,22) – 200(P/F,10%,44)
– 200(P/F,10%,44)
= –35,000 + 4800(P/F,10%,44) – 300(P/A,10%,44) – 5200(P/F,10%,22)
= –35,000 + 4800(0.0154) – 300(9.8461) – 5200(0.1228)
= $–38,519

Not very sensitive.

3. In all cases, plan A has the best PW.

4. Use SOLVER to find the breakeven values of P_A for the three MARR values of 8%, 10%, and 15% per year.

For MARR = 8%, the SOLVER screen is below.



Breakeven values are:

MARR	Breakeven P_A
8%	\$–33,556
10	–33,734
15	–33,975

The P_A breakeven value is not sensitive, but all three outcomes are over 3X the \$10,000 estimated first cost for plan A.

Case Study Solution

1. Let x = weighting per factor

   Since there are 6 factors and one (environmental considerations) is to have a weighting that is double the others, its weighting is 2x. Thus,

$$2x + x + x + x + x + x = 100$$
$$7x = 100$$
$$x = 14.3\%$$

   Therefore, the environmental weighting is 2(14.3), or 28.6%

2.

Alt ID	Ability to Supply Area	Relative Cost	Engineering Feasibility	Institutional Issues	Environmental Considerations	Lead-Time Requirement	Total
1A	5(0.2)	4(0.2)	3(0.15)	4(0.15)	5(0.15)	3(0.15)	4.1
3	5(0.2)	4(0.2)	4(0.15)	3(0.15)	4(0.15)	3(0.15)	3.9
4	4(0.2)	4(0.2)	3(0.15)	3(0.15)	4(0.15)	3(0.15)	3.6
8	1(0.2)	2(0.2)	1(0.15)	1(0.15)	3(0.15)	4(0.15)	2.0
12	5(0.2)	5(0.2)	4(0.15)	1(0.15)	3(0.15)	1(0.15)	3.4

   Therefore, the top three are the same as before: 1A, 3, and 4

3. For alternative 4 to be as economically attractive as alternative 3, its total annual cost would have to be the same as that of alternative 3, which is \$3,881,879. Thus, if P_4 is the capital investment,

$$3{,}881{,}879 = P_4(A/P, 8\%, 20) + 1{,}063{,}449$$
$$3{,}881{,}879 = P_4(0.10185) + 1{,}063{,}449$$
$$P_4 = \$27{,}672{,}361$$

$$\text{Decrease} = 29{,}000{,}000 – 27{,}672{,}361$$
$$= \$1{,}327{,}639 \text{ or } 4.58\%$$

4. Household cost at 100% = 3,952,959(1/12)(1/4980)(1/1)
   = \$66.15

$$\text{Decrease} = 69.63 – 66.15$$
$$= \$3.48 \text{ or } 5\%$$

5. (a) Sensitivity analysis of M&O and number of households.

Alternative	Estimate	M&O, $/year	Number of households	Total annual cost, $/year	Household cost, $/month
1A	Pessimistic	1,071,023	4980	3,963,563	69.82
	Most likely	1,060,419	5080	3,952,959	68.25
	Optimistic	1,049,815	5230	3,942,355	66.12
3	Pessimistic	910,475	4980	3,925,235	69.40
	Most likely	867,119	5080	3,881,879	67.03
	Optimistic	867,119	5230	3,881,879	**65.10**
4	Pessimistic	1,084,718	4980	4,038,368	71.13
	Most likely	1,063,449	5080	4,017,099	69.37
	Optimistic	957,104	5230	3,910,754	65.59

Conclusion: Alternative 3 – optimistic is the best.

(b) Let x be the number of households. Set alternative 4 – optimistic cost equal to $65.10.

$$(3{,}910{,}754)/12(0.95)(x) = \$65.10$$
$$x = 5270$$

This is an increase of only 40 households.