Chapter 19

More on Variation and Decision Making Under Risk

Solutions to Problems

19.1 (a) Continuous (assumed) and uncertain – no chance statements made.
(b) Discrete and risk – plot units vs. chance as a continuous straight line between 50 and 55 units.
(c) 2 variables: first is discrete and certain at $400; second is continuous for ≥ $400, but uncertain (at this point). More data needed to assign any probabilities.
(d) Discrete variable with risk; rain at 20%, snow at 30%, other at 50%.

19.2 Needed or assumed information to be able to calculate an expected value:
1. Treat output as discrete or continuous variable .
2. If discrete, center points on cells, e.g., 800, 1500, and 2200 units per week.
3. Probability estimates for < 1000 and /or > 2000 units per week.

19.3 (a) N is discrete since only specific values are mentioned; i is continuous from 0 to 12.
(b) The P(N), F(N), P(i) and F(i) are calculated below.

N	0	1	2	3	4
P(N)	.12	.56	.26	.03	.03
F(N)	.12	.68	.94	.97	1.00

i	0-2	2-4	4-6	6-8	8-10	10-12
P(i)	.22	.10	.12	.42	.08	.06
F(i)	.22	.32	.44	.86	.94	1.00

(c) $P(N = 1 \text{ or } 2) = P(N = 1) + P(N = 2)$

$= 0.56 + 0.26 = 0.82$

or

$F(N \le 2) – F(N \le 0) = 0.94 – 0.12 = 0.82$

$P(N \ge 3) = P(N = 3) + P(N \ge 4) = 0.06$

(d) $P(7\% \le i \le 11\%) = P(6.01 \le i \le 12)$

$= 0.42 + 0.08 + 0.06 = 0.56$

or

$F(i \le 12\%) - F(i \le 6\%) = 1.00 - 0.44$
$= 0.56$

19.4 (a)

$	0	2	5	10	100
F($)	.91	.955	.98	.993	1.000

The variable $ is discrete, so plot $ versus F($).

(b) $E(\$) = \sum \$P(\$) = 0.91(0) + ... + 0.007(100)$
$= 0 + 0.09 + 0.125 + 0.13 + 0.7$
$= \$1.045$

(c) $2.000 - 1.045 = 0.955$
Long-term income is 95.5 cents per ticket

19.5 (a) $P(N) = (0.5)^N$ $\quad N = 1,2,3,...$

N	1	2	3	4	5	etc.
P(N)	0.5	0.25	0.125	0.0625	0.03125	
F(N)	0.5	0.75	0.875	0.9375	0.96875	

Plot P(N) and F(N); N is discrete.

P(L) is triangular like the distribution in Figure 19-5 with the mode at 5.

$$f(\text{mode}) = f(M) = \frac{2}{5-2} = \frac{2}{3}$$

$$F(\text{mode}) = F(M) = \frac{5-2}{5-2} = 1$$

(b) $P(N = 1, 2 \text{ or } 3) = F(N \le 3) = 0.875$

19.6 First cost, P

P_P = first cost to purchase
P_L = first cost to lease

Use the uniform distribution relations in Equation [19.3] and plot.

$f(P_P) = 1/(25{,}000–20{,}000) = 0.0002$

$f(P_L) = 1/(2000–1800) = 0.005$

Salvage value, S

S_P is triangular with mode at \$2500.

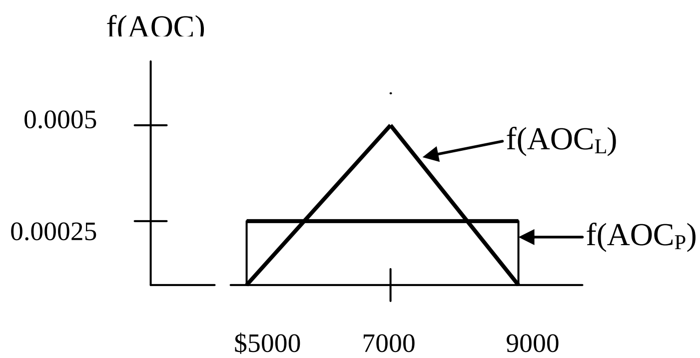
The $f(S_P)$ is symmetric around \$2500.
$f(M) = f(2500) = 2/(1000) = 0.002$ is the probability at \$2500.

There is no S_L distribution

AOC

$f(AOC_P) = 1/(9000–5000) = 0.00025$

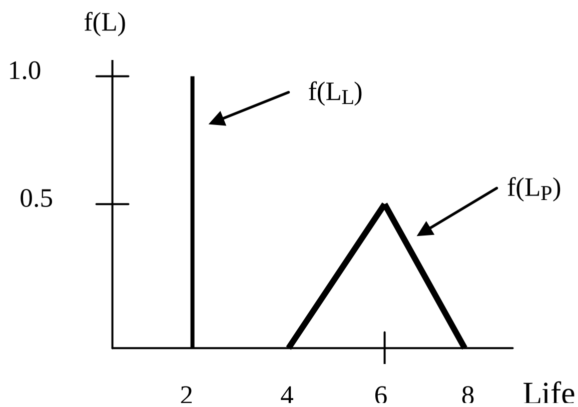
$f(AOC_L)$ is triangular with:
$f(7000) = 2/(9000–5000) = 0.0005$



Life, L

$f(L_P)$ is triangular with mode at 6.
$f(6) = 2/(8-4) = 0.5$

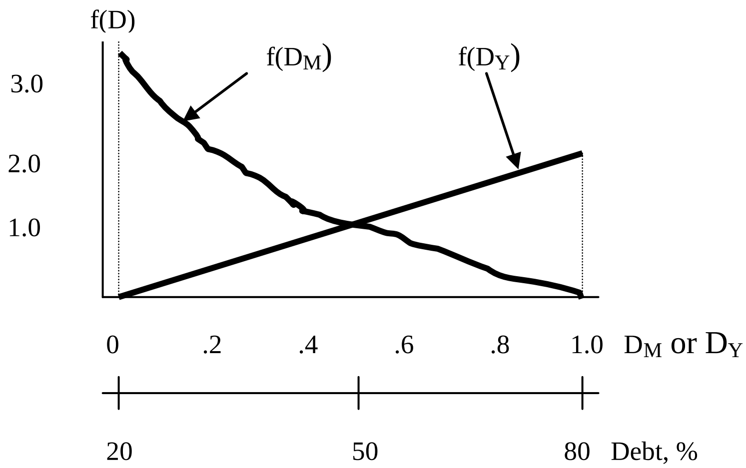
The value L_L is certain at 2 years.



19.7 (a) Determine several values of D_M and D_Y and plot.

D_M or D_Y	$f(D_M)$	$f(D_Y)$
0.0	3.00	0.0
0.2	1.92	0.4
0.4	1.08	0.8
0.6	0.48	1.2
0.8	0.12	1.6
1.0	0.00	2.0

$f(D_M)$ is a decreasing power curve and $f(D_Y)$ is linear.



(b) Probability is larger that M (mature) companies have a lower debt percentage and that Y (young) companies have a higher debt percentage.

19.8 (a)

X_i	1	2	3	6	9	10
$F(X_i)$	0.2	0.4	0.6	0.7	0.9	1.0

(b) $P(6 \le X \le 10) = F(10) - F(3) = 1.0 - 0.6 = 0.4$

or

$P(X = 6, 9 \text{ or } 10) = 0.1 + 0.2 + 0.1 = 0.4$

$P(X = 4, 5 \text{ or } 6) = F(6) - F(3) = 0.7 - 0.6 = 0.1$

(c) $P(X = 7 \text{ or } 8) = F(8) - F(6) = 0.7 - 0.7 = 0.0$

No sample values in the 50 have X = 7 or 8. A larger sample is needed to observe all values of X.

19.9 Plot the $F(X_i)$ from Problem 19.8 (a), assign the RN values, use Table 19.2 to obtain 25 sample X values; calculate the sample $P(X_i)$ values and compare them to the stated probabilities in 19.8.

(Instructor note: Point out to students that it is not correct to develop the sample $F(X_i)$ from another sample where some discrete variable values are omitted).

19.10 (a)

X	0	.2	.4	.6	.8	1.0
F(X)	0	.04	.16	.36	.64	1.00

Take X and p values from the graph. Some samples are:

RN	X	p
18	.42	7.10%
59	.76	8.80
31	.57	7.85
29	.52	7.60

(b) Use the sample mean for the average p value. Our sample of 30 had p = 6.3375%; yours will vary depending on the RNs from Table 19.2.

19.11 Use the steps in Section 19.3. As an illustration, assume the probabilities that are assigned by a student are:

$$P(G = g) = \begin{cases} 0.30 & G=A \\ 0.40 & G=B \\ 0.20 & G=C \\ 0.10 & G=D \\ 0.00 & G=F \\ 0.00 & G=I \end{cases}$$

Steps 1 and 2: The F(G) and RN assignment are:

$$F(G = g) = \begin{cases} 0.30 & G=A & 00\text{-}29 \\ 0.70 & G=B & 30\text{-}69 \\ 0.90 & G=C & 70\text{-}89 \\ 1.00 & G=D & 90\text{-}99 \\ 1.00 & G=F & \text{--} \\ 1.00 & G=I & \text{--} \end{cases}$$

(RNs column: 00-29, 30-69, 70-89, 90-99, --, --)

Steps 3 and 4: Develop a scheme for selecting the RNs from Table 19-2. Assume you want 25 values. For example, if RN_1 = 39, the value of G is B. Repeat for sample of 25 grades.

Step 5: Count the number of grades A through D, calculate the probability of each as count/25, and plot the probability distribution for grades A through I. Compare these probabilities with P(G = g) above.

19.12 (a) When RAND( ) was used for 100 values in column A of an Excel spreadsheet, the function AVERAGE(A1:A100) resulted in 0.50750658; very close to 0.5.

RANDBETWEEN(0,1) generates only integer values of 0 or 1. For one sample of 100, the average was 0.48; in another it was exactly 0.50.

(b) For the RAND results, count the number of values in each cell to determine how close it is to 10.

19.13 (a) Use Equations [19.9] and [19.12] or the spreadsheet functions AVERAGE and STDEV.

Cell, X_i	f_i	X_i^2	f_iX_i	$f_iX_i^2$
600	6	360,000	3,600	2,160,000
800	10	640,000	8,000	6,400,000
1000	9	1,000,000	9,000	9,000,000
1200	15	1,440,000	18,000	21,600,000
1400	28	1,960,000	39,200	54,880,000
1600	15	2,560,000	24,000	38,400,000
1800	7	3,240,000	12,600	22,680,000
2000	10	4,000,000	20,000	40,000,000
	100		134,400	195,120,000

AVERAGE: Xbar = 134,400/100 = 1344.00

STDEV: $s^2 = \left[\frac{195,120,000}{99} - \frac{100}{99}(1344)^2 \right]^2$

$= (146,327.27)^2$

$= 382.53$

(b) Xbar ± 2s is 1344.00 ± 2(382.53) = 578.94 and 2109.06
All values are in the ±2s range.

(c) Plot X versus f. Indicate Xbar and the range Xbar ± 2s on it.

19.14 (a) Convert P(X) data to frequency values to determine s.

X	P(X)	XP(X)	f	X^2	fX^2
1	.2	.2	10	1	10
2	.2	.4	10	4	40
3	.2	.6	10	9	90
6	.1	.6	5	36	180
9	.2	1.8	10	81	810
10	.1	1.0	5	100	500
		4.6			1630

Sample average: Xbar = 4.6

Sample variance: $s^2 = \frac{1630}{49} - \frac{50}{49}(4.6)^2 = 11.67$

$s = 3.42$

(b) Xbar ± 1s is 4.6 ± 3.42 = 1.18 and 8.02
25 values, or 50%, are in this range.

Xbar ± 2s is 4.6 ± 6.84 = –2.24 and 11.44
All 50 values, or 100%, are in this range.

19.15 (a) Use Equations [19.15] and [19.16]. Substitute Y for D_Y.

$f(Y) = 2Y$

$$E(Y) = \int_0^1 (Y)2Y dy$$
$$= \left[\frac{2Y^3}{3}\right]_0^1$$
$$= 2/3 - 0 = 2/3$$

$$Var(Y) = \int_0^1 (Y^2)2Y dy - [E(Y)]^2$$
$$= \left[\frac{2Y^4}{4}\right]_0^1 - (2/3)^2$$
$$Var(Y) = \frac{2}{4} - 0 - \frac{4}{9}$$

$$= 1/18 = 0.05556$$

$$\sigma = (0.05556)^{0.5} = 0.236$$

(b) $E(Y) \pm 2\sigma$ is 0.667 ± 0.472 = 0.195 and 1.139

Take the integral from 0.195 to 1.0 only since the variable's upper limit is 1.0.

$$P(0.195 \le Y \le 1.0) = \int_{0.195}^{1} 2Y dy$$

$$= Y^2 \Big|_{0.195}^{1}$$

$$= 1 - 0.038 = 0.962 \qquad (96.2\%)$$

19.16 (a) Use Equations [19.15] and [19.16]. Substitute M for D_M.

$$E(M) = \int_0^1 (M)\, 3\,(1 - M)^2 dm$$

$$= 3 \int_0^1 (M - 2M^2 + M^3) dm$$

$$= 3 \left[\frac{M^2}{2} - \frac{2M^3}{3} + \frac{M^4}{4}\right]_0^1$$

$$= \frac{3}{2} - 2 + \frac{3}{4} = \frac{6 - 8 + 3}{4} = \frac{1}{4} = 0.25$$

$$Var(M) = \int_0^1 (M^2)\, 3\,(1 - M)^2 dm - [E(M)]^2$$

$$= 3 \int_0^1 (M^2 - 2M^3 + M^4) dm - (1/4)^2$$

$$= 3 \left[\frac{M^3}{3} - \frac{M^4}{2} + \frac{M^5}{5}\right]_0^1 - 1/16$$

$$= 1 - 3/2 + 3/5 - 1/16$$
$$= (80 - 120 + 48 - 5)/80$$
$$= 3/80 = 0.0375$$

$$\sigma = (0.0375)^{0.5} = 0.1936$$

(b) $E(M) \pm 2\sigma$ is $0.25 \pm 2(0.1936) = -0.1372$ and 0.6372

Use the relation defined in Problem 19.15 to take the integral from 0 to 0.6372.

$$P(0 \le M \le 0.6372) = \int_0^{0.6372} 3(1 - M)^2 \, dm$$

$$= 3 \int_0^{0.6372} (1 - 2M + M^2) dm$$

$$= 3 \left[M - M^2 + 1/3\, M^3 \right]_0^{0.6372}$$

$$= 3 \left[0.6372 - (0.6372)^2 + 1/3\, (0.6372)^3 \right]$$

$$= 0.952 \qquad\qquad (95.2\%)$$

19.17 Use Eq. [19.8] where $P(N) = (0.5)^N$

$E(N) = 1(.5) + 2(.25) + 3(.125) + 4(0.625) + 5(.03125) + 6(.015625) + 7(.0078125)$
$+ 8(.003906) + 9(.001953) + 10(.0009766) + ..$
$= 1.99+$

E(N) can be calculated for as many N values as you wish. The limit to the series $N(0.5)^N$ is 2.0, the correct answer.

19.18 $E(Y) = 3(1/3) + 7(1/4) + 10(1/3) + 12(1/12)$
$= 1 + 1.75 + 3.333 + 1$
$= 7.083$

$Var(Y) = \sum Y^2P(Y) - [E(Y)]^2$
$= 3^2(1/3) + 7^2(1/4) + 10^2(1/3) + 12^2(1/12) - (7.083)^2$
$= 60.583 - 50.169$
$= 10.414$

$\sigma = 3.227$

$E(Y) \pm 1\sigma$ is $7.083 \pm 3.227 = 3.856$ and 10.310

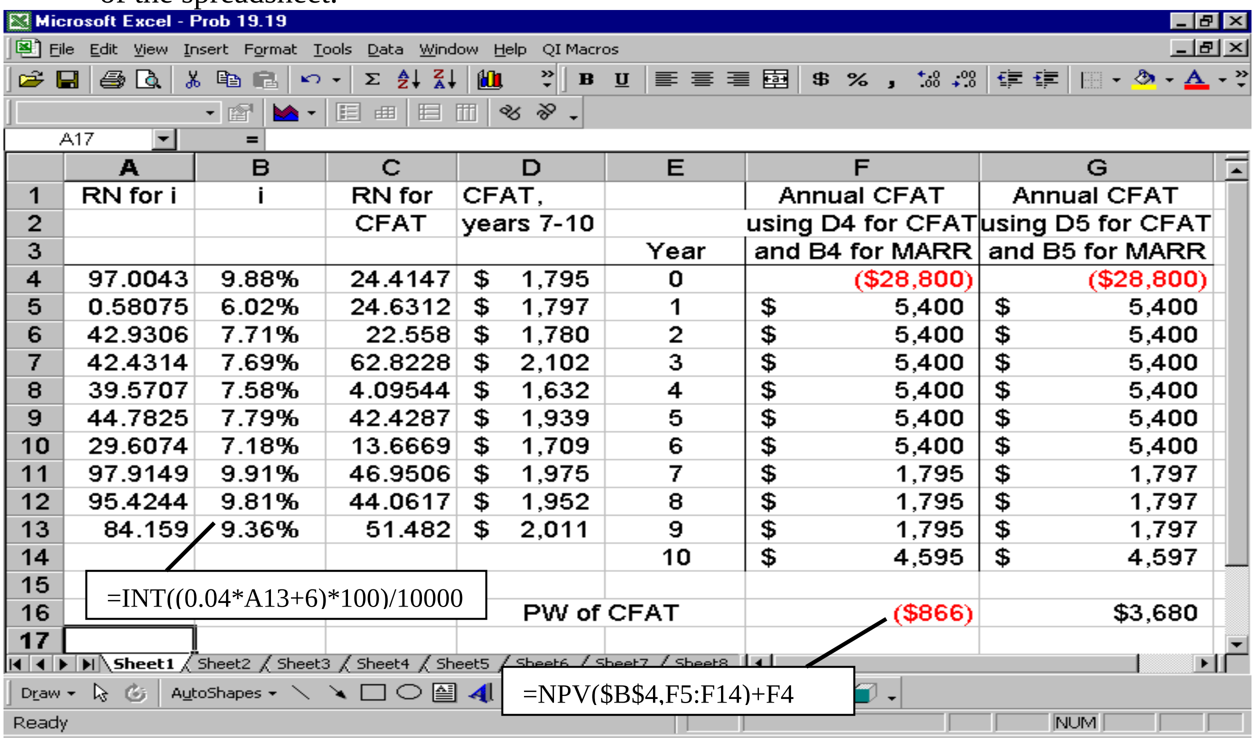
19.19 Using a spreadsheet, the steps in Sec. 19.5 are applied.

1. CFAT given for years 0 through 6.
2. i varies between 6% and 10%.
   CFAT for years 7-10 varies between \$1600 and \$2400.
3. Uniform for both i and CFAT values.

19.19 (cont)

4. Set up a spreadsheet. The example below has the following relations:

   Col A: =RAND ( )* 100 to generate random numbers from 0-100.
   Col B, cell B4: =INT((.04*A4+6) *100)/10000 converts the RN to i from 0.06 to 0.10. The % designation changes it to an interest rate between 6% and 10%.
   Col C: = RAND( )* 100
   Col D, cell D4: =INT (8*C4+1600) to convert to a CFAT between $1600 and $2400.
   Ten samples of i and CFAT for years 7-10 are shown below in columns B and D of the spreadsheet.



Microsoft Excel - Prob 19.19

	A	B	C	D	E	F	G
1	RN for i	i	RN for	CFAT,		Annual CFAT	Annual CFAT
2			CFAT	years 7-10		using D4 for CFAT	using D5 for CFAT
3					Year	and B4 for MARR	and B5 for MARR
4	97.0043	9.88%	24.4147	$ 1,795	0	($28,800)	($28,800)
5	0.58075	6.02%	24.6312	$ 1,797	1	$ 5,400	$ 5,400
6	42.9306	7.71%	22.558	$ 1,780	2	$ 5,400	$ 5,400
7	42.4314	7.69%	62.8228	$ 2,102	3	$ 5,400	$ 5,400
8	39.5707	7.58%	4.09544	$ 1,632	4	$ 5,400	$ 5,400
9	44.7825	7.79%	42.4287	$ 1,939	5	$ 5,400	$ 5,400
10	29.6074	7.18%	13.6669	$ 1,709	6	$ 5,400	$ 5,400
11	97.9149	9.91%	46.9506	$ 1,975	7	$ 1,795	$ 1,797
12	95.4244	9.81%	44.0617	$ 1,952	8	$ 1,795	$ 1,797
13	84.159	9.36%	51.482	$ 2,011	9	$ 1,795	$ 1,797
14					10	$ 4,595	$ 4,597
15							
16				PW of CFAT		($866)	$3,680
17							

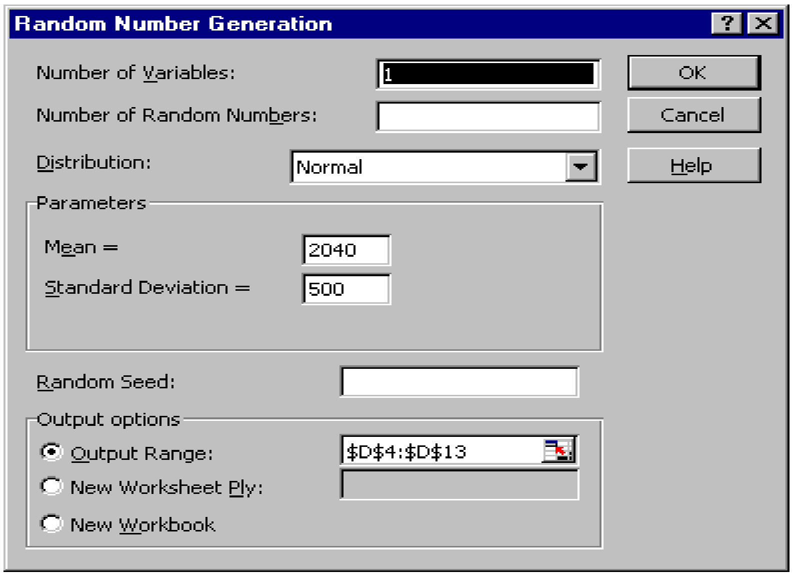
5. Columns F and G give two of the CFAT sequences, for example only, using rows 4 and 5 random number generations. The entry for cells F11 through F13 is =D4 and cell F14 is =D4+2800, where S = $2800. The PW values are obtained using the spreadsheet NPV function. The value PW = $-866 results from the i value in B4 (i = 9.88%) and PW = $3680 results from applying the MARR in B5 (i = 6.02%).
6. Plot the PW values for as large a sample as desired. Or, following the logic of Figure 19-13, a spreadsheet relation can count the + and – PW values, with Xbar and s calculated for the sample.

7. Conclusion:

For certainty, accept the plan since PW = $2966 exceeds zero at an MARR of 7% per year.

For risk, the result depends on the preponderance of positive PW values from the simulation, and the distribution of PW obtained in step 6.

19.20 Use the spreadsheet Random Number Generator (RNG) on the tools toolbar to generate CFAT values in column D from a normal distribution with μ = $2040 and σ = $500. The RNG screen image is shown below. (This tool may not be available on all spreadsheets.)



19.20 (cont)

	A	B	C	D	E	F	G
1	RN for i	i	RN for	CFAT,		Annual CFAT	Annual CFAT
2			CFAT	years 7-10		using D4 for CFAT	using D5 for CFAT
3					Year	and B4 for MARR	and B5 for MARR
4	68.67539	8.74%	23.82629	2348	0	($28,800)	($28,800)
5	82.13034	9.28%	23.18529	1284	1	$ 5,400	$ 5,400
6	3.610742	6.14%	33.13977	2422	2	$ 5,400	$ 5,400
7	82.22524	9.28%	86.80954	2454	3	$ 5,400	$ 5,400
8	55.16774	8.20%	77.58184	2603	4	$ 5,400	$ 5,400
9	23.5219	6.94%	52.37264	2939	5	$ 5,400	$ 5,400
10	29.72799	7.18%	72.8421	1477	6	$ 5,400	$ 5,400
11	19.07978	6.76%	8.014663	2181	7	$ 2,348	$ 1,284
12	79.72004	9.18%	3.419809	2393	8	$ 2,348	$ 1,284
13	51.65328	8.06%	7.080597	1983	9	$ 2,348	$ 1,284
14					10	$ 5,148	$ 4,084
15							
16				PW of CFAT		$1,452	($1,197)

The spreadsheet above is the same as that in Problem 19.19, except that CFAT values in column D for years 7 through 10 are generated using the RNG for the normal distribution described above. The decision to accept the plan uses the same logic as that described in Problem 19.19.

Extended Exercise Solution

This simulation is left to the student and the instructor. The same 7-step procedure from Section 19.5 applied in Problems 19.19 and 19.20 is used to set up the RNG for the cash flow values AOC and S, and the alternative life n for each alternative. The distributions given in the statement of the exercise are defined using the RNG.

For each of the 50-sample cash flow series, calculate the AW value for each alternative. To obtain a final answer of which alternative is the best to accept, it is recommended that the number of positive and negative AW values be counted as they are generated. Then the alternative with the most positive AW values indicates which one to accept. Of course, due to the RNG generation of AOC, S and n values, this decision may vary from one simulation run to the next.