Chapter 3
Combining Factors

Solutions to Problems

3.1 P = 100,000(260)(P/A,10%,8)(P/F,10%,2)
= 26,000,000(5.3349)(0.8264)
= $114.628 million

3.2 P = 50,000(56)(P/A,8%,4)(P/F,8%,1)
= 2,800,000(3.3121)(0.9259)
= $8.587 million

3.3 P = 80(2000)(P/A,18%,3) + 100(2500)(P/A,18%,5)(P/F,18%,3)
= 160,000(2.1743) + 250,000(3.1272)(0.6086)
= $823,691

3.4 P = 100,000(P/A,15%,3) + 200,000(P/A,15%,2)(P/F,15%,3)
= 100,000(2.2832) + 200,000(1.6257)(0.6575)
= $442,100

3.5 P = 150,000 + 150,000(P/A,10%,5)
= 150,000 + 150,000(3.7908)
= $718,620

3.6 P = 3500(P/A,10%,3) + 5000(P/A,10%,7)(P/F,10%,3)
= 3500(2.4869) + 5000(4.8684)(0.7513)
= $26,992

3.7 A = [0.701(5.4)(P/A,20%,2) + 0.701(6.1)(P/A,20%,2)((P/F,20%,2)](A/P,20%,4)
= [3.7854(1.5278) + 4.2761(1.5278)(0.6944)](0.38629)
= $3.986 billion

3.8 A = 4000 + 1000(F/A,10%,4)(A/F,10%,7)
= 4000 + 1000(4.6410)(0.10541)
= $4489.21

3.9 A = 20,000(P/A,8%,4)(A/F,8%,14)
= 20,000(3.3121)(0.04130)
= $2735.79

3.10 A = 8000(A/P,10%,10) + 600
= 8000(0.16275) + 600
= $1902

3.11 A = 20,000(F/P,8%,1)(A/P,8%,8)
= 20,000(1.08)(0.17401)
= $3758.62

3.12 A = 10,000(F/A,8%,26)(A/P,8%,30)
= 10,000(79.9544)(0.08883)
= $71,023

3.13 A = 15,000(F/A,8%,9)(A/F,8%,10)
= 15,000(12.4876)(0.06903)
= $12,930

3.14 A = 80,000(A/P,10%,5) + 80,000
= 80,000(0.26380) + 80,000
= $101,104

3.15 A = 5000(A/P,6%,5) + 1,000,000(0.15)(0.75)
= 5000(0.2374) + 112,500
= $113,687

3.16 A = [20,000(F/A,8%,11) + 8000(F/A,8%,7)](A/F,8%,10)
= [20,000(16.6455) + 8000(8.9228)]{0.06903)
= $27,908

3.17 A = 600(A/P,12%,5) + 4000(P/A,12%,4)(A/P,12%,5)
= 600(0.27741) + 4000(3.0373)(0.27741)
= $3536.76

3.18 F = 10,000(F/A,15%,21)
= 10,000(118.8101)
= $1,188,101

3.19 100,000 = A(F/A,7%,5)(F/P,7%,10)
100,000 = A(5.7507)(1.9672)
A = $8839.56

3.20 F = 9000(F/P,8%,11) + 600(F/A,8%,11) + 100(F/A,8%,5)
= 9000(2.3316) + 600(16.6455) + 100(5.8666)
= $31,558

3.21 Worth in year 5 = -9000(F/P,12%,5) + 3000(P/A,12%,9)
= -9000(1.7623) + 3000(5.3282)
= $123.90

3.22 Amt, year 5 = 1000(F/A,12%,4)(F/P,12%,2) + 2000(P/A,12%,7)(P/F,12%,1)
= 1000(4.7793)(1.2544) + 2000(4.5638)(0.8929)
= $14,145

3.23 A = [10,000(F/P,12%,3) + 25,000](A/P,12%,7)
= [10,000(1.4049) + 25,000](0.21912)
= $8556.42

3.24 Cost of the ranch is P = 500(3000) = $1,500,000.
1,500,000 = x + 2x(P/F,8%,3)
1,500,000 = x + 2x(0.7938)
x = $579,688

3.25 Move unknown deposits to year –1, amortize using A/P, and set equal to $10,000.
x(F/A,10%,2)(F/P,10%,19)(A/P,10%,15) = 10,000
x(2.1000)(6.1159)(0.13147) = 10,000
x = $5922.34

3.26 350,000(P/F,15%,3) = 20,000(F/A,15%,5) + x
350,000(0.6575) = 20,000(6.7424) + x
x = $95,277

3.27 Move all cash flows to year 9.
0 = -800(F/A,14%,2)(F/P,14%,8) + 700(F/P,14%,7) + 700(F/P,14%,4)
–950(F/A,14%,2)(F/P,14%,1) + x – 800(P/A,14%,3)
0 = -800(2.14)2.8526) + 700(2.5023) + 700(1.6890)
–950(2.14)(1.14) + x – 800(2.3216)
x = $6124.64

3.28 Find P at t = 0 and then convert to A.
P = 5000 + 5000(P/A,12%,3) + 3000(P/A,12%,3)(P/F,12%,3)
+ 1000(P/A,12%,2)(P/F,12%,6)
= 5000 + 5000(2.4018) + 3000(2.4018)(0.7118)
+ 1000(1.6901)(0.5066)
= $22,994

A = 22,994(A/P,12%,8)
= 22,994(0.20130)
= $4628.69

3.29 F = 2500(F/A,12%,8)(F/P,12%,1) – 1000(F/A,12%,3)(F/P,12%,2)
= 2500(12.2997)(1.12) – 1000(3.3744)(1.2544)
= $30,206

3.30 $15{,}000 = 2000 + 2000(P/A,15\%,3) + 1000(P/A,15\%,3)(P/F,15\%,3) + x(P/F,15\%,7)$
$15{,}000 = 2000 + 2000(2.2832) + 1000(2.2832)(0.6575) + x(0.3759)$
$x = \$18{,}442$

3.31 Amt, year 3 $= 900(F/A,16\%,4) + 3000(P/A,16\%,2) - 1500(P/F,16\%,3)$
$+ 500(P/A,16\%,2)(P/F,16\%,3)$
$= 900(5.0665) + 3000(1.6052) - 1500(0.6407)$
$+ 500(1.6052)(0.6407)$
$= \$8928.63$

3.32 $A = 5000(A/P,12\%,7) + 3500 + 1500(F/A,12\%,4)(A/F,12\%,7)$
$= 5000(0.21912) + 3500 + 1500(4.7793)(0.09912)$
$= \$5306.19$

3.33 $20{,}000 = 2000(F/A,15\%,2)(F/P,15\%,7) + x(F/A,15\%,7) + 1000(P/A,15\%,3)$
$20{,}000 = 2000(2.1500)(2.6600) + x(11.0668) + 1000(2.2832)$
$x = \$567.35$

3.34 $P = [4{,}100{,}000(P/A,6\%,22) - 50{,}000(P/G,6\%,22)](P/F,6\%,3)$
$+ 4{,}100{,}000(P/A,6\%,3)$
$= [4{,}100{,}000(12.0416) - 50{,}000(98.9412](0.8396)$
$+ 4{,}100{,}000(2.6730)$
$= \$48{,}257{,}271$

3.35 $P = [2{,}800{,}000(P/A,12\%,7) + 100{,}000(P/G,12\%,7) + 2{,}800{,}000](P/F,12\%,1)$
$= [2{,}800{,}000(4.5638) + 100{,}000(11.6443) + 2{,}800{,}000](0.8929)$
$= \$14{,}949{,}887$

3.36 P for maintenance $= [11{,}500(F/A,10\%,2) + 11{,}500(P/A,10\%,8)$
$+ 1000(P/G,10\%,8)](P/F,10\%,2)$
$= [11{,}500(2.10) + 11{,}500(5.3349) + 1000(16.0287)](0.8264)$
$= \$83{,}904$
P for accidents $= 250{,}000(P/A,10\%,10)$
$= 250{,}000(6.1446)$
$= \$1{,}536{,}150$
Total savings $= 83{,}904 + 1{,}536{,}150$
$= \$1{,}620{,}054$
Build overpass

3.37 Find P at t = 0, then convert to A.
$P = [22{,}000(P/A,12\%,4) + 1000(P/G,12\%,4) + 22{,}000](P/F,12\%,1)$
$= [22{,}000(3.0373) + 1000(4.1273) + 22{,}000](0.8929)$
$= \$82{,}993$

A = 82,993(A/P,12%,5)
= 82,993(0.27741)
= $23,023

3.38 First find P and then convert to F.

P = -10,000 + [4000 + 3000(P/A,10%,6) + 1000(P/G,10%,6)
– 7000(P/F,10%,4)](P/F,10%,1)
= -10,000 + [4000 + 3000(4.3553) + 1000(9.6842)
– 7000(0.6830)](0.9091)
= $9972

F = 9972(F/P,10%,7)
= 9972(1.9487)
= $19,432

3.39 Find P in year 0 and then convert to A.

P = 4000 + 4000(P/A,15%,3) – 1000(P/G,15%,3) + [(6000(P/A,15%,4)
+2000(P/G,15%,4)](P/F,15%,3)
= 4000 + 4000(2.2832) – 1000(2.0712) + [(6000(2.8550)
+2000(3.7864)](0.6575)
= $27,303.69

A = 27,303.69(A/P,15%,7)
= 27,303.69(0.24036)
= $6563

3.40 40,000 = x(P/A,10%,2) + (x + 2000)(P/A,10%,3)(P/F,10%,2)
40,000 = x(1.7355) + (x + 2000)(2.4869)(0.8264)
3.79067x = 35,889.65
x = $9467.89 (size of first two payments)

3.41 11,000 = 200 + 300(P/A,12%,9) + 100(P/G,12%,9) – 500(P/F,12%,3)
+ x(P/F,12%,3)
11,000 = 200 + 300(5.3282) + 100(17.3563) – 500(0.7118) + x(0.7118)
x = $10,989

3.42 (a) In billions

P in yr 1 = $-13(2.73) + 5.3\{[1 - (1 + 0.09)^{10}/ (1 + 0.15)^{10}]/(0.15 - 0.09)\}$
= -35.49 + 5.3(6.914)
= $1.1542 billion

P in yr 0 = 1.1542(P/F,15%,1)
= 1.1542(0.8696)
= $1.004 billion

3.43 Find P in year –1; then find A in years 0-5.

P_g in yr 2 = $(5)(4000)\{[1 - (1 + 0.08)^{18}/(1 + 0.10)^{18}]/(0.10 - 0.08)\}$
= 20,000(14.0640)
= $281,280

P in yr –1 = 281,280(P/F,10%,3) + 20,000(P/A,10%,3)
= 281,280(0.7513) + 20,000(2.4869)
= $261,064

A = 261,064(A/P,10%,6)
= 261,064(0.22961)
= $59,943

3.44 Find P in year –1 and then move forward 1 year

P_{-1}= $20,000\{[1 – (1 + 0.05)^{11}/(1 + 0.14)^{11}]/(0.14 – 0.05)\}$.
= 20,000(6.6145)
= $132,290

P = 132,290(F/P,14%,1)
= 132,290(1.14)
= $150,811

3.45 P = 29,000 + 13,000(P/A,10%,3) + 13,000[7/(1 + 0.10)](P/F,10%,3)
= 29,000 + 13,000(2.4869) + 82,727(0.7513)
= $123,483

3.46 Find P in year –1 and then move to year 0.

P (yr –1) = $15,000\{[1 – (1 + 0.10)^5/(1 + 0.16)^5]/(0.16 – 0.10)\}$
= 15,000(3.8869)
= $58,304

P = 58,304(F/P,16%,1)
= 58,304(1.16)
= $67,632

3.47 Find P in year –1 and then move to year 5.

P (yr –1) = 210,000[6/(1 + 0.08)]
= 210,000(0.92593)
= $1,166,667

F = 1,166,667(F/P,8%,6)
= 1,166,667(1.5869)
= $1,851,383

3.48 P = [2000(P/A,12%,6) – 200(P/G,12%,6)](F/P,12%,1)
= [2000(4.1114) – 200(8.9302](1.12)
= $7209.17

3.49 P = 5000 + 1000(P/A,12%,4) + [1000(P/A,12%,7) – 100(P/G,12%,7)](P/F,12%,4)
= 5000 + 1000(3.0373) + [1000(4.5638) – 100(11.6443)](0.6355)
= $10,198

3.50 Find P in year 0 and then convert to A.
P = 2000 + 2000(P/A,10%,4) + [2500(P/A,10%,6) – 100(P/G,10%,6)](P/F,10%,4)
= 2000 + 2000(3.1699) + [2500(4.3553) – 100(9.6842)](0.6830)
= $15,115

A = 15,115(A/P,10%,10)
= 15,115(0.16275)
= $2459.97

3.51 20,000 = 5000 + 4500(P/A,8%,n) – 500(P/G,8%,n)
Solve for n by trial and error:
Try n = 5: $15,000 > $14,281
Try n = 6: $15,000 < $15,541
By interpolation, n = 5.6 years

3.52 P = 2000 + 1800(P/A,15%,5) – 200(P/G,15%,5)
= 2000 + 1800(3.3522) – 200(5.7751)
= $6878.94

3.53 F = [5000(P/A,10%,6) – 200(P/G,10%,6)](F/P,10%,6)
= [5000(4.3553) – 200(9.6842)](1.7716)
= $35,148

FE Review Solutions

3.54 x = 4000(P/A,10%,5)(P/F,10%,1)
= 4000(3.7908)(0.9091)
= $13,785
Answer is (d)

3.55 P = 7 + 7(P/A,4%,25)
= $116.3547 million
Answer is (c)

3.56 Answer is (d)

3.57 Size of first deposit = $1250/(1 + 0.05)^3$
= $1079.80
Answer is (d)

3.58 Balance = 10,000(F/P,10%,2) – 3000(F/A,10%,2)
= 10,000(1.21) – 3000(2.10)
= $5800
Answer is (b)

3.59 1000 = A(F/A,10%,5)(A/P,10%,20)
1000 = A(6.1051)(0.11746)
A = $1394.50
Answer is (a)

3.60 First find P and then convert to A.
P = 1000(P/A,10%,5) + 2000(P/A,10%,5)(P/F,10%,5)
= 1000(3.7908) + 2000(3.7908)(0.6209)
= $8498.22

A = 8498.22(A/P,10%,10)
= 8498.22(0.16275)
= $1383.08
Answer is (c)

3.61 100,000 = A(F/A,10%,4)(F/P,10%,1)
100,000 = A(4.6410)(1.10)
A = $19,588
Answer is (a)

3.62 F = [1000 + 1500(P/A,10%,10) + 500(P/G,10%,10](F/P,10%,10)
= [1000 + 1500(6.1446) + 500(22.8913](2.5937)
= $56,186
Answer is (d)

3.63 F = 5000(F/P,10%,10) + 7000(F/P,10%,8) + 2000(F/A,10%,5)
= 5000(2.5937) + 7000(2.1438) + 2000(6.1051)
= $40,185
Answer is (b)

Extended Exercise Solution

Solution by Hand

Cash flows for purchases at g = –25% start in year 0 at $4 million. Cash flows for parks development at G = $100,000 start in year 4 at $550,000. All cash flow signs in the solution are +.

	Cash flow	
Year	Land	Parks
0	$4,000,000	
1	3,000,000	
2	2,250,000	
3	1,678,000	
4	1,265.625	$550,000
5	949,219	650,000
6		750,000

1. Find P for all project funds (in $ million)

   $P = 4 + 3(P/F,7\%,1) + \ldots + 0.750(P/F,7\%,6)$
   $= 13.1716$ ($13,171,600)

   Amount to raise in years 1 and 2:

   $A = (13.1716 – 3.0)(A/P,7\%,2)$
   $= (10.1716)(0.55309)$
   $= 5.6258$ ($5,625,800 per year)

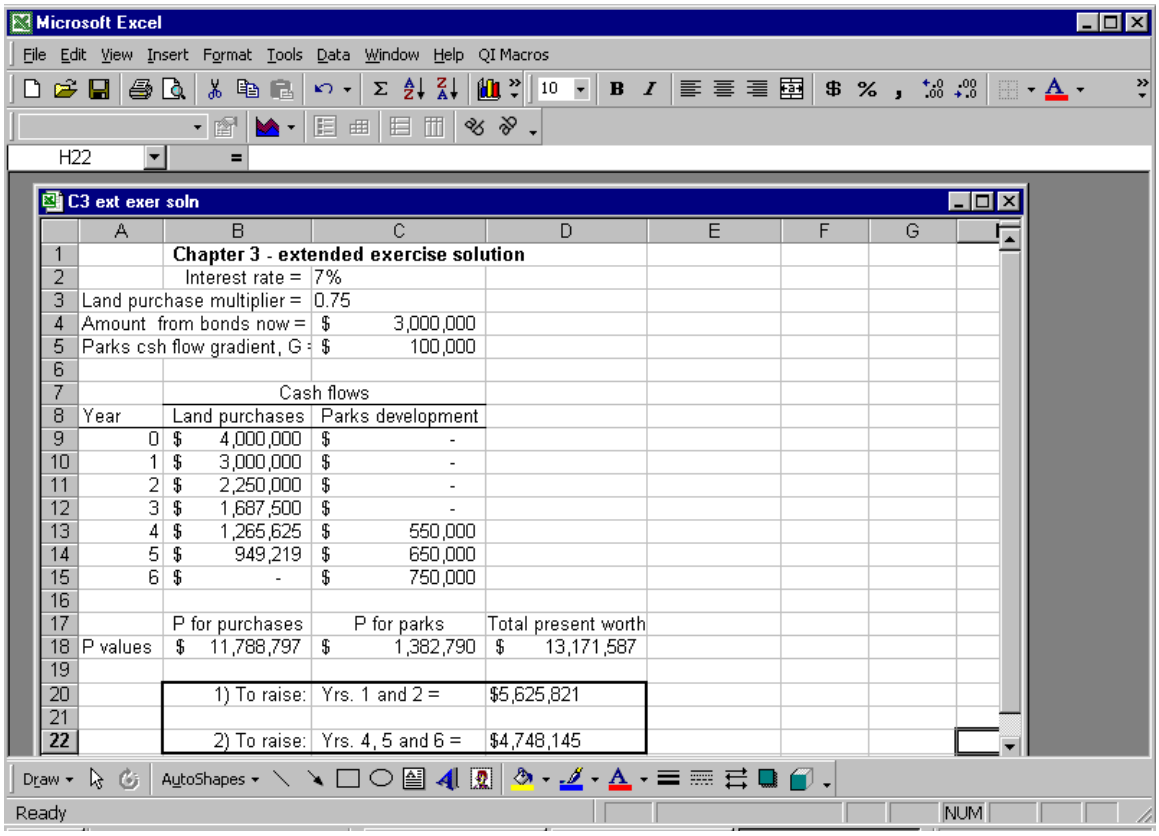
2. Find remaining project fund needs in year 3, then find the A for the next 3 years (years 4, 5, and 6):

   $F_3 = (13.1716 – 3.0)(F/P,7\%,3)$
   $= (10.1716)(1.2250)$
   $= 12.46019$

   $A = 12.46019(A/P,7\%,3)$
   $= 12.46019(0.38105)$
   $= 4.748$ ($4,748,000 per year)

Extended Exercise Solution

Solution by computer



	A	B	C	D	E	F	G
1		**Chapter 3 - extended exercise solution**					
2		Interest rate =	7%				
3	Land purchase multiplier =		0.75				
4	Amount from bonds now =		$ 3,000,000				
5	Parks csh flow gradient, G =		$ 100,000				
6							
7		Cash flows					
8	Year	Land purchases	Parks development				
9	0	$ 4,000,000	$ -				
10	1	$ 3,000,000	$ -				
11	2	$ 2,250,000	$ -				
12	3	$ 1,687,500	$ -				
13	4	$ 1,265,625	$ 550,000				
14	5	$ 949,219	$ 650,000				
15	6	$ -	$ 750,000				
16							
17		P for purchases	P for parks	Total present worth			
18	P values	$ 11,788,797	$ 1,382,790	$ 13,171,587			
19							
20		1) To raise:	Yrs. 1 and 2 =	$5,625,821			
21							
22		2) To raise:	Yrs. 4, 5 and 6 =	$4,748,145			